

Q Find $\mathcal{L}^{-1}\left(\frac{1}{s^2(s^2+4)}\right)$ using Partial Fraction method

Solution

$$\mathcal{L}^{-1}\frac{1}{s^2(s^2+4)} = \mathcal{L}^{-1}\left(\frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+4}\right)$$

$$= \mathcal{L}^{-1}\frac{A(s^2+4) + Bs(s^2+4) + s^2(Cs+D)}{s^2(s^2+4)} =$$

$$= \mathcal{L}^{-1}\frac{As^2+4A + Bs^3+4Bs + Cs^3+Ds^2}{s^2(s^2+4)}$$

$$= \mathcal{L}^{-1}\frac{s^3(B+C) + s^2(A+D) + 4Bs + 4A}{s^2(s^2+4)}$$

$$\therefore 1 = s^3(B+C) + s^2(A+D) + 4Bs + 4A$$

$$B+C=0$$

$$A+D=0$$

$$4B=0 \Rightarrow \boxed{B=0} \Rightarrow \boxed{C=0}$$

$$4A=1 \Rightarrow \boxed{A=\frac{1}{4}} \Rightarrow \boxed{D=-\frac{1}{4}}$$

$$\therefore \mathcal{L}^{-1}\left(\frac{\frac{1}{4}}{s^2} + \frac{0}{s} + \frac{0s - \frac{1}{4}}{s^2+4}\right)$$

$$\begin{aligned} &= \frac{1}{4}\mathcal{L}^{-1}\frac{1}{s^2} - \frac{1}{4}\mathcal{L}^{-1}\frac{1}{s^2+4} \times \frac{2}{2} = \frac{1}{4}\mathcal{L}^{-1}\frac{1}{s^2} - \frac{1}{8}\mathcal{L}^{-1}\frac{2}{s^2+4} \\ &= \frac{1}{4}t - \frac{1}{8}\sin 2t = f(t) \end{aligned}$$

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$$\textcircled{Q} \mathcal{L}^{-1} \frac{1}{s^3(s^2+1)} = \mathcal{L}^{-1} \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{s^2+1}$$

$$= \mathcal{L}^{-1} \frac{A(s^2+1) + Bs(s^2+1) + Cs^2(s^2+1) + s^3(Ds+E)}{s^3(s^2+1)}$$

$$= \mathcal{L}^{-1} \frac{As^2 + A + Bs^3 + Bs + Cs^4 + Cs^2 + Ds^4 + Es^3}{s^3(s^2+1)}$$

$$= \mathcal{L}^{-1} \frac{s^4(C+D) + s^3(B+E) + s^2(A+C) + Bs + A}{s^3(s^2+1)}$$

$$\therefore C+D=0$$

$$B+E=0$$

$$A+C=0$$

$$\boxed{B=0} \Rightarrow \boxed{E=0}$$

$$\boxed{A=1} \Rightarrow \boxed{C=-1} \Rightarrow \boxed{D=1}$$

$$= \mathcal{L}^{-1} \frac{1}{s^3} + \frac{0}{s^2} + \frac{-1}{s} + \frac{s+0}{s^2+1}$$

$$= \mathcal{L}^{-1} \frac{1}{s^3} - \frac{1}{s} + \frac{s}{s^2+1} = \frac{1}{2}t^2 - 1 + \cos t = f(t)$$

$$\mathcal{L}^{-1} \frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} = \mathcal{L}^{-1} \frac{As + B}{s^2 + 2s + 2} + \frac{Cs + D}{s^2 + 2s + 5}$$

$$= \mathcal{L}^{-1} \frac{(As + B)(s^2 + 2s + 5) + (Cs + D)(s^2 + 2s + 2)}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

$$= \mathcal{L}^{-1} \frac{As^3 + 2As^2 + 5As + Bs^2 + 2Bs + 5B + Cs^3 + 2Cs^2 + 2Cs + Ds^2 + 2Ds + 2D}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

$$= \mathcal{L}^{-1} \frac{s^3(A+C) + s^2(2A+B+2C+D) + s(5A+2B+2C+2D) + (5B+2D)}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

$$A+C=0 \quad \text{--- (1)} \Rightarrow \boxed{A=-C} \text{ put in (2) and (3)}$$

$$2A+B+2C+D=1 \quad \text{--- (2)}$$

$$5A+2B+2C+2D=2 \quad \text{--- (3)}$$

$$5B+2D=3 \quad \text{--- (4)} \Rightarrow \boxed{B = \frac{3-2D}{5}} \text{ put in (2)}$$

$$\text{from (2)} \quad -2C + \frac{3-2D}{5} + 2C + D = 1 \Rightarrow \boxed{D = \frac{2}{3}}$$

$$\boxed{B = \frac{1}{3}}$$

$$(2) \rightarrow 2A + \frac{1}{3} + 2C + \frac{2}{3} = 1$$

$$(3) \rightarrow 5A + \frac{2}{3} + 2C + \frac{4}{3} = 2$$

$$\therefore 2A + 2C = 0$$

$$5A + 2C = 0 \Rightarrow \boxed{A=C=0}$$

$$\Rightarrow \mathcal{L}^{-1} \left(\frac{1}{3} \frac{1}{s^2+2s+2} + \frac{2}{3} \frac{1}{s^2+2s+5} \right)$$

$$= \mathcal{L}^{-1} \frac{1}{3} + \mathcal{L}^{-1} \frac{2}{3}$$

$$(s^2+2s+1)^{-1+2} \quad (s^2+2s+1)^{-1+5}$$

$$= \frac{1}{3} \mathcal{L}^{-1} \frac{1}{(s+1)^2+1} + \frac{2}{3} \mathcal{L}^{-1} \frac{1}{(s+1)^2+4}$$

$$= \frac{1}{3} e^{-t} \mathcal{L}^{-1} \frac{1}{s^2+1} + \frac{2}{3} e^{-t} \mathcal{L}^{-1} \frac{1}{s^2+4}$$

$$= \frac{1}{3} e^{-t} \mathcal{L}^{-1} \frac{1}{s^2+1} + \frac{1}{3} e^{-t} \mathcal{L}^{-1} \frac{2}{s^2+4}$$

$$= \frac{1}{3} e^{-t} \sin t + \frac{1}{3} e^{-t} \sin 2t$$

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"Problems"

1- $\frac{s^2 + 6s + 6}{(s+1)(s^2 + 4s + 3)}$

11- $\frac{s}{(s+1)^5}$

2- $\frac{12s - 28}{(s-5)(s+3)(s-1)}$

12- $\frac{3s - 14}{s^2 - 4s + 8}$

3- $\frac{3s^2 + 8s + 9}{(s+3)(s+1)^2}$

13- $\frac{8s + 20}{s^2 - 12s + 32}$

4- $\frac{s^3 + 4s^2 + 4s - 5}{(s+3)(s+1)^3}$

5- $\frac{2s^2}{(s^2+1)(s-1)^2}$

6- $\frac{s}{s^2 + 6s + 1}$

7- $\frac{s+1}{s^2 + s + 1}$

8- $\left\{ \frac{12}{4-3s} \right\}$

9- $\frac{1}{2s-5}$

10- $\left\{ \frac{3(s^2-1)^2}{2s^5} + \frac{4s-18}{9-s^2} \right\}$

$$1) \mathcal{L}^{-1} \frac{s^2 + 6s + 6}{(s+1)(s^2 + 4s + 3)} = \mathcal{L}^{-1} \frac{s^2 + 6s + 6}{(s+1)(s+3)(s+1)} = \mathcal{L}^{-1} \frac{s^2 + 6s + 6}{(s+1)^2(s+3)}$$

$$\mathcal{L}^{-1} \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+3)} + \dots$$

$$4) \mathcal{L}^{-1} \frac{s^3 + 4s^2 + 4s - 5}{(s+3)(s+1)^3} = \mathcal{L}^{-1} \frac{A}{(s+1)^3} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)} + \frac{D}{s+3}$$

$$5) \mathcal{L}^{-1} \frac{2s^2}{(s^2+1)(s-1)^2} = \mathcal{L}^{-1} \frac{As+B}{s^2+1} + \frac{C}{(s-1)^2} + \frac{D}{(s-1)}$$

$$6) \mathcal{L}^{-1} \frac{s}{s^2 + 6s + 1} \quad \left(\begin{array}{l} \text{اكمال مربع} \\ \text{مربع اقل من صفر} \end{array} \right)$$

$$\mathcal{L}^{-1} \frac{s}{(s^2 + 6s + 9) + 1 - 9} = \mathcal{L}^{-1} \frac{s}{(s+3)^2 - 8} \Rightarrow \mathcal{L}^{-1} \frac{s+3-3}{(s+3)^2 - 8}$$

$$\mathcal{L}^{-1} \frac{s+3}{(s+3)^2 - 8} - \frac{3}{(s+3)^2 - 8}$$

$$12) \mathcal{L}^{-1} \frac{s}{(s+1)^5} \Rightarrow \mathcal{L}^{-1} \frac{s+1-1}{(s+1)^5} \Rightarrow \mathcal{L}^{-1} \frac{s+1}{(s+1)^5} - \frac{1}{(s+1)^5}$$

$$\mathcal{L}^{-1} \frac{1}{(s+1)^4} - \frac{1}{(s+1)^5}$$