

(4)

Ex Solve the initial value problem by laplace trans. $y'' - y' - 2y = e^{2t}$ $y(0)=0, y'(0)=1$

Solu: take laplace trans. on both sides of the equation we get

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - \mathcal{L}\{2y\} = \mathcal{L}\{e^{2t}\}$$

$$s^2 Y(s) - 1 - s Y(s) - 2Y(s) = \frac{1}{s-2}$$

Solve it for Y

$$(s^2 - s - 2) Y(s) = \frac{1}{s-2} + 1 \Rightarrow Y(s) = \frac{s-1}{(s-2)^2(s+1)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

take laplace inverse

$$\mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \left\{ \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2} \right\}$$

$$= \frac{A(s-2)^2 + B(s+1)(s-2) + C(s+1)}{(s-2)^2(s+1)}$$

$$s-1 = As^2 - 4As + 4A + Bs^2 - Bs - 2B + Cs + C$$

$$\left. \begin{array}{l} A+B=0 \\ -4A-B+C=1 \\ 4A-2B+C=-1 \end{array} \right\} \Rightarrow A = -\frac{2}{9}, B = \frac{2}{9}, C = \frac{1}{3}$$

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Solve
w:

$$= \mathcal{L}^{-1} \left\{ \frac{-\frac{2}{9}}{s+1} + \frac{\frac{2}{9}}{s-2} + \frac{\frac{1}{3}}{(s-2)^2} \right\}$$

$$= y(t) = -\frac{2}{9} e^{-t} + \frac{2}{9} e^{2t} + \frac{1}{3} t e^{2t}$$

Ex $y'' - 5y' + 6y = 0$ Solve the ordinary diff equation using laplace trans. where $y(0) = 2$
 $y'(0) = 2$

Solution

Take laplace: $\mathcal{L}\{y'' - 5y' + 6y\} = \mathcal{L}(0)$

$$s^2 Y(s) - 2s - 2 - 5[sY(s) - 2] + 6Y(s) = 0$$

$$Y(s)(s^2 - 5s + 6) = 2s - 8 \Rightarrow Y(s) = \frac{2s - 8}{s^2 - 5s + 6}$$

$$Y(s) = \frac{2s - 8}{(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s-2}$$

$$= \mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \left\{ \frac{A}{s-3} + \frac{B}{s-2} \right\}, \quad A = -2$$

$$B = 4$$

$$= y(t) = \mathcal{L}^{-1} \frac{-2}{s-3} + \mathcal{L}^{-1} \frac{4}{s-2}$$

$$y(t) = -2 e^{3t} + 4 e^{2t}$$

(6)

Solve the (I.v.p.) $\ddot{y}(t) + y(t) = 3\sin 2t$

with $y(0)=1$, $\dot{y}(0)=-2$

Solution let $Y(s) = \mathcal{L}\{y(t)\}$

take laplace on both sides

$$\mathcal{L}\ddot{y}(t) + \mathcal{L}y(t) = 3\mathcal{L}\sin 2t$$

$$s^2 Y(s) - sy(0) - \dot{y}(0) + Y(s) = \frac{6}{s^2+4} \Rightarrow s^2 Y(s) - s - 2 + Y(s) = \frac{6}{s^2+4}$$

$$Y(s)(s^2+1) = \frac{s^3 - 2s^2 + 4s - 2}{(s^2+4)} \Rightarrow Y(s) = \frac{s^3 - 2s^2 + 4s - 2}{(s^2+4)(s^2+1)}$$

$$Y(s) = \frac{As+B}{(s^2+4)} + \frac{Cs+D}{(s^2+1)} = \frac{(As+B)(s^2+1) + (Cs+D)(s^2+4)}{(s^2+4)(s^2+1)}$$

$$s^3 - 2s^2 + 4s - 2 = As^3 + As + Bs^2 + B + Cs^3 + 4Cs + Ds^2 + 4D$$

$$C=1, A=0, D=0, B=-2$$

$$y(t) = \mathcal{L}^{-1} \frac{-2}{s^2+4} + \mathcal{L}^{-1} \frac{s}{s^2+1}$$

$$y(t) = -\sin 2t + \cos t$$

$$A+C=1$$

$$B+D=-2$$

$$A+4C=4$$

$$B+4D=-2$$

Ex Solve the initial value problem by Laplace transform $y'' - y' - 2y = e^{2t}$ $y(0)=0, y'(0)=1$

Solu take Laplace for both sides

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} - \mathcal{L}\{2y\} = \mathcal{L}\{e^{2t}\}$$

$$s^2 Y(s) - s y(0) - y'(0) - (s Y(s) - y(0)) - 2 Y(s) = \frac{1}{s-2}$$

$$s^2 Y(s) - 1 - s Y(s) - 2 Y(s) = \frac{1}{s-2}$$

$$Y(s) [s^2 - s - 2] = \frac{1}{s-2} + 1$$

$$Y(s) [(s-2)(s+1)] = \frac{1+s-2}{(s-2)} = \frac{s-1}{(s-2)}$$

$$\Rightarrow Y(s) = \frac{s-1}{(s-2)^2(s+1)} \Rightarrow Y(s) = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

$$= \frac{A(s-2)^2 + B(s+1)(s-2) + C(s+1)}{(s-2)^2(s+1)}$$

$$s-1 = AS^2 - 4AS + 4A + BS^2 - BS - 2B + CS + C$$

$$A+B=0 \quad \text{--- (1)} \Rightarrow A=-B \quad \text{put in (2) (3)}$$

$$-4A - B + C = 1 \quad \text{--- (2)}$$

$$\Rightarrow 4B - B + C = 1$$

$$4A - 2B + C = -1 \quad \text{--- (3)} \Rightarrow -4B - 2B + C = -1$$

$$\begin{array}{r} 3B + C = 1 \\ -6B + C = -1 \end{array} \quad \text{eq 4}$$

$$9B = 2 \Rightarrow \boxed{B = \frac{2}{9}} \quad A = -\frac{2}{9} \quad C = \frac{1}{3}$$

$$\therefore \mathcal{L}^{-1} \left[\frac{-\frac{2}{9}}{s+1} + \frac{\frac{2}{9}}{s-2} + \frac{\frac{1}{3}}{(s-2)^2} \right]$$

$$y(t) = -\frac{2}{9} e^{-t} + \frac{2}{9} e^{2t} + \frac{1}{3} e^{2t} \mathcal{L}^{-1} \left[\frac{1}{s^2} \right]$$

$$y(t) = -\frac{2}{9} e^{-t} + \frac{2}{9} e^{2t} + \frac{1}{3} e^{2t} t$$

$$\underline{\text{Ex}} \quad y'' - 5y' + 6y = 0$$

$$y(0) = 2$$

$$y'(0) = 2$$

Solution

$$\mathcal{L}\{y'' - 5y' + 6y\} = \mathcal{L}\{0\}$$

$$s^2 Y(s) - 5sY(s) + 6Y(s) = 0$$

$$s^2 Y(s) - 2s - 2 - 5sY(s) + 10 + 6Y(s) = 0$$

$$Y(s)[s^2 - 5s + 6] = 2s - 8$$

$$Y(s)[(s-3)(s-2)] = 2s - 8$$

$$Y(s) = \frac{2s-8}{(s-3)(s-2)} = \frac{A}{s-3} + \frac{B}{s-2}$$

$$\mathcal{L}^{-1} Y(s) = \mathcal{L}^{-1} \left\{ \frac{A}{s-3} + \frac{B}{s-2} \right\}$$

$$A = \frac{2s-8}{(s-3)(s-2)} \Big|_{s=3} = \frac{-2}{1} = \boxed{-2 = A}$$

$$B = \frac{2s-8}{(s-3)(s-2)} \Big|_{s=2} = \frac{-4}{-1} = 4 = B$$

$$y(t) = \mathcal{L}^{-1} \left\{ \frac{-2}{s-3} + \frac{4}{s-2} \right\} = -2e^{3t} + 4e^{2t}$$