

(6)

(7)

Ex Solve:

H.W

$$x + 2y + z = 8$$

$$(x=1, y=2, z=3)$$

$$3x + 4y + 2z = 17$$

$$6y - 20 = -5x - z \quad (\text{four decimal places})$$

$$\underline{\text{Ex}} \quad 4.44x - 9.94y + 2.41z = 5.36$$

$$8.24x + 2.02y - 2.18z = 9.34$$

$$0.93x + 3.86y + 11.66z = 2.57$$

Solution

$$Aib = \begin{pmatrix} 4.44 & -9.94 & 2.41 & 5.36 \\ 8.24 & 2.02 & -2.18 & 9.34 \\ 0.93 & 3.86 & 11.66 & 2.57 \end{pmatrix}$$

$$\begin{pmatrix} 8.24 & 2.02 & -2.18 & 9.34 \\ 4.44 & -9.94 & 2.41 & 5.36 \\ 0.93 & 3.86 & 11.66 & 2.57 \end{pmatrix}$$

$$\begin{pmatrix} 8.24 & 2.02 & -2.18 & 9.34 \\ 0 & -11.0284 & 3.5846 & 0.3272 \\ 0 & 3.632 & 11.906 & 1.5158 \end{pmatrix} \Rightarrow \begin{pmatrix} 8.24 & 2.02 & -2.18 & 9 \\ 0 & -11.0284 & 3.5846 & 0.3 \\ 0 & 0 & 13.0865 & 1.623 \end{pmatrix}$$

$$z = 0.124, y = 0.0276, x = 1.159$$

7

Ex! Use Gaussian elimination to solve the sample system of linear eq.s

$$2x_1 - 3x_2 + 4x_3 = 2$$

$$4x_1 + x_2 + 2x_3 = 2$$

$$x_1 - x_2 + 3x_3 = 3$$

Solution:

$$A|b \Rightarrow \left(\begin{array}{ccc|c} 2 & -3 & 4 & 2 \\ 4 & 1 & 2 & 2 \\ 1 & -1 & 3 & 3 \end{array} \right) R_1 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 4 & 1 & 2 & 2 \\ 2 & -3 & 4 & 2 \end{array} \right) \begin{array}{l} R_2 = R_2 - 4R_1 \\ R_3 = R_3 - 2R_1 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 5 & -10 & -10 \\ 0 & -1 & -2 & -4 \end{array} \right)$$

$$R_2 = \frac{1}{5}R_2 \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -2 & -4 \end{array} \right) R_3 = R_3 + R_2$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -4 & -6 \end{array} \right)$$

$$\Rightarrow x_3 = \frac{-6}{-4} = \frac{3}{2}, \quad x_2 = 1, \quad x_1 = -\frac{1}{2}$$

(8)

Example Solve the system using Gaussian elimination method:

$$2x_1 + 4x_2 + 4x_3 + 2x_4 = 16$$

$$4x_1 + 8x_2 + 6x_3 + 8x_4 = 32$$

$$14x_1 + 29x_2 + 32x_3 + 16x_4 = 112$$

$$10x_1 + 17x_2 + 10x_3 + 2x_4 = 28$$

Solution: Form the matrix $(A:b)$

$$\left(\begin{array}{cccc|c} 2 & 4 & 4 & 2 & 16 \\ 4 & 8 & 6 & 8 & 32 \\ 14 & 29 & 32 & 16 & 112 \\ 10 & 17 & 10 & 2 & 28 \end{array} \right) \begin{array}{l} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 7R_1 \\ R_4 = R_4 - 5R_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 2 & 4 & 4 & 2 & 16 \\ 0 & 0 & -2 & 4 & 0 \\ 0 & 1 & 4 & 2 & 0 \\ 0 & -3 & -10 & -8 & -52 \end{array} \right) \begin{array}{l} \text{note} \\ \text{the main diagonal elements} \\ \neq 0 \end{array}$$

$$\text{So } R_2 \leftrightarrow R_3 \left(\begin{array}{cccc|c} 2 & 4 & 4 & 2 & 16 \\ 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & -2 & 4 & 0 \\ 0 & -3 & -10 & -8 & -52 \end{array} \right) \begin{array}{l} \\ R_4 = R_4 + 3R_2 \end{array}$$

$$\left(\begin{array}{cccc|c} 2 & 4 & 4 & 2 & 16 \\ 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & -2 & 4 & 0 \\ 0 & 0 & 2 & -2 & -52 \end{array} \right) \begin{array}{l} \\ R_4 = R_4 + R_3 \end{array} \left(\begin{array}{cccc|c} 2 & 4 & 4 & 2 & 16 \\ 0 & 1 & 4 & 2 & 0 \\ 0 & 0 & -2 & 4 & 0 \\ 0 & 0 & 0 & 2 & -52 \end{array} \right)$$

$$\therefore x_4 = -26, x_3 = -52, x_2 = 260, x_1 = -382$$

(9)

Ex 2

$$2x_1 - 3x_2 + 4x_3 = 2$$

$$4x_1 + x_2 + 2x_3 = 2$$

$$x_1 - x_2 + 3x_3 = 3$$

Solve the System using
Gaussian-elimination method

Solution

$$(A|b) \Rightarrow \left(\begin{array}{ccc|c} 2 & -3 & 4 & 2 \\ 4 & 1 & 2 & 2 \\ 1 & -1 & 3 & 3 \end{array} \right) R_1 \Leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 4 & 1 & 2 & 2 \\ 2 & -3 & 4 & 2 \end{array} \right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 5 & -10 & -10 \\ 0 & -1 & -2 & -4 \end{array} \right)$$

$$R_2 = R_2 - (4/1)R_1$$

$$= R_2 - 4R_1$$

$$= 4 - 4(1) = 0$$

$$= 1 - 4(-1) = 5$$

$$= 2 - 4(3) = -10$$

$$= 2 - 4(3) = -10$$

$$R_3 = R_3 - (2/1)R_1$$

$$= R_3 - 2R_1$$

$$= 2 - 2(1) = 0$$

$$= -3 - 2(-1) = -1$$

$$= 4 - 2(3) = -2$$

$$= 2 - 2(3) = -4$$

$$\Rightarrow R_2 = \frac{1}{5}R_2 \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -2 & -4 \end{array} \right)$$

$$\Rightarrow R_3 = R_3 - (-1/1)R_2$$

$$R_3 = R_3 + R_2$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 3 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -4 & -6 \end{array} \right) \begin{array}{l} x_1 - x_2 + 3x_3 = 3 \\ x_2 - 2x_3 = -2 \\ -4x_3 = -6 \end{array}$$

$$x_3 = \frac{-6}{-4} = \frac{3}{2}, \quad x_2 = 1, \quad x_1 = -\frac{1}{2}$$