

Ex Solve

$$\begin{aligned} 2x_2 + x_4 &= 0 \\ 2x_1 + 2x_2 + 3x_3 + 2x_4 &= -2 \\ 4x_1 - 3x_2 + x_4 &= -7 \\ 6x_1 + x_2 - 6x_3 - 5x_4 &= 6 \end{aligned}$$

Using Gauss elimination (four decimal places)

Solution

* Form the matrix (A, b)

$$\left(\begin{array}{cccc|c} 0 & 2 & 0 & 1 & 0 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 6 & 1 & -6 & -5 & 6 \end{array} \right)$$

* we note $a_{11} = 0 \Rightarrow$ (division by zero error)

* to avoid this error we will use

(The partial pivoting) by rearrange the

equations (rows) we put " a_{ii} " the largest element regardless the sign

$$\left(\begin{array}{cccc|c} 6 & 1 & -6 & -5 & 6 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right)$$

(1)

$$\left(\begin{array}{cccc|c} 6 & 1 & -6 & -5 & 6 \\ 2 & 2 & 3 & 2 & -2 \\ 4 & -3 & 0 & 1 & -7 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right)$$

Put $a_{ii}=0$ $i=2,3,4$

using $R_i = R_i - (a_{i1}/a_{11}) R_1$

$$i=2 \quad (a_{21}=0) \\ (a_{21}/a_{11}) = \frac{2}{6} \Rightarrow R_2 = R_2 - \left(\frac{a_{21}}{a_{11}}\right) R_1$$

$$2 - \frac{2}{6} 6 = 0$$

$$2 - \frac{2}{6} (1) = \frac{10}{6} = 1.6666$$

$$\left| \begin{array}{l} -2 - \frac{2}{6} (6) = -4 \end{array} \right.$$

$$3 - \frac{2}{6} (-6) = 5$$

$$2 - \frac{2}{6} (-5) = \frac{22}{6} = 3.6666$$

$$i=3 \quad (a_{31}=0)$$

$$R_3 = R_3 - \frac{a_{31}}{a_{11}} R_1 \Rightarrow R_3 = R_3 - \frac{4}{6} R_1$$

$$4 - \frac{4}{6} 6 = 0$$

$$-3 - \frac{4}{6} (1) = -\frac{22}{6} = -3.6666$$

$$0 - \frac{4}{6} (-6) = 4$$

$$1 - \frac{4}{6} (-5) = \frac{26}{6} = 4.3333$$

$$-7 - \frac{4}{6} (6) = -11$$

$$\left(\begin{array}{cccc|c} 6 & 1 & -6 & -5 & 6 \\ 0 & \frac{10}{6} & 5 & \frac{22}{6} & -4 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} & -11 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right)$$

using partial

pivoting

$R_3 \leftrightarrow R_2$

(2)

$$\begin{pmatrix} 6 & 1 & -6 & -5 & 6 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} & -11 \\ 0 & \frac{10}{6} & 5 & -\frac{22}{6} & -4 \\ 0 & 2 & 0 & -1 & 0 \end{pmatrix} \quad \begin{array}{l} a_{12} = 0 \quad i=3,4 \\ R_i = R_i - (a_{i2}/a_{22})R_2 \end{array}$$

$$a_{32}/a_{22} \Rightarrow R_3 = R_3 - \frac{a_{32}}{a_{22}} R_2 \quad \left[\frac{a_{32}}{a_{22}} \quad \frac{\frac{10}{6}}{-\frac{22}{6}} = -\frac{10}{22} \right]$$

$$0 + \frac{10}{22} \cdot 0 = 0 \quad R_3 - (-\frac{10}{22})R_2 = R_3 + \frac{10}{22}R_2$$

$$\frac{10}{6} + \frac{10}{22}(-\frac{22}{6}) = 0$$

$$5 + \frac{10}{22}(4) = \frac{150}{22} \approx 6.8181$$

$$\frac{22}{6} + \frac{10}{22}(\frac{26}{6}) = \frac{744}{132} \approx 5.6363 \rightarrow \frac{372}{66} = \frac{186}{33} = \frac{62}{11}$$

$$-4 + \frac{10}{22}(-11) = -9$$

$$a_{42}/a_{22} \Rightarrow R_4 = R_4 - \frac{a_{42}}{a_{22}} R_2 \Rightarrow R_4 - (-\frac{2}{22})R_2 = R_4 + \frac{12}{22}R_2$$

$$0 + \frac{12}{22} \cdot 0 = 0$$

$$2 + \frac{12}{22}(-\frac{22}{6}) = 0$$

$$0 + \frac{12}{22}(4) = \frac{48}{22} \approx 2.1818$$

$$1 + \frac{12}{22}(\frac{26}{6}) = \frac{444}{132} \approx 3.3636$$

$$0 + \frac{12}{22}(-11) = -6$$

$$(R_4 = R_4 - \frac{a_{43}}{a_{33}} R_3)$$

(3)

$$\begin{pmatrix} 6 & 1 & -6 & -5 & 6 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} & -11 \\ 0 & 0 & \frac{150}{22} & \frac{744}{132} & -9 \\ 0 & 0 & \frac{48}{22} & \frac{444}{132} & -6 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 6 & 1 & -6 & -5 & 6 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} & -11 \\ 0 & 0 & \frac{150}{22} & \frac{744}{132} & -9 \\ 0 & 0 & \frac{48}{22} & \frac{444}{132} & -6 \end{array} \right)$$

$$R_4 = R_4 - \frac{48}{22} R_3 = R_4 - \frac{48}{22} \times \frac{22}{150} R_3 = R_4 - \frac{24}{75} R_3$$

$$\frac{48}{22} - \frac{24}{75} \frac{150}{22} = 0$$

$$\frac{444}{132} - \frac{24}{75} \frac{744}{132} = \frac{15444}{9900} = 1.56$$

$$-6 - \frac{24}{75} (-9) = \frac{-450 + 216}{75} = \frac{-234}{75}$$

$$\left(\begin{array}{cccc|c} 6 & 1 & -6 & -5 & 6 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} & -11 \\ 0 & 0 & \frac{150}{22} & \frac{744}{132} & -9 \\ 0 & 0 & 0 & \frac{15444}{9900} & \frac{-234}{75} \end{array} \right)$$

This is an upper triangular matrix now using back substitution to find x_4

$$x_n = a_{n,n+1} / a_{nn} \Rightarrow x_4 = \frac{a_{45}}{a_{44}}$$

$$x_i = (a_{i,n+1} - \sum_{j=i+1}^n a_{ij} x_j) / a_{ii}$$

to find x_i

$$x_3 = (a_{35} - a_{34} x_4) / a_{33}$$

$$x_2 = [a_{25} - (a_{23} x_3 + a_{24} x_4)] / a_{22}$$

$$x_1 = [a_{15} - (a_{12} x_2 + a_{13} x_3 + a_{14} x_4)] / a_{11}$$

(4)

$$\begin{pmatrix} 6 & 1 & -6 & -5 \\ 0 & -\frac{22}{6} & 4 & \frac{26}{6} \\ 0 & 0 & \frac{150}{22} & \frac{744}{132} \\ 0 & 0 & 0 & \frac{15444}{9900} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6 \\ -11 \\ -9 \\ -\frac{234}{75} \end{pmatrix}$$

$$0x_1 + 0x_2 + 0x_3 + \frac{15444}{9900}x_4 = -\frac{234}{75} \Rightarrow x_4 = \frac{-234}{\frac{15444}{75}}$$

$$\boxed{x_4 = -2}$$

$$\therefore 0x_1 + 0x_2 + \frac{150}{22}x_3 + \frac{744}{132}(-2) = -9 \Rightarrow \boxed{x_3 = \frac{1}{3}}$$

$$0x_1 - \frac{22}{6}x_2 + 4\left(\frac{1}{3}\right) + \frac{26}{6}(-2) = -11 \Rightarrow \boxed{x_2 = 1}$$

$$6x_1 + 1(1) - 6\left(\frac{1}{3}\right) - 5(-2) = 6 \Rightarrow \boxed{x_1 = -\frac{1}{2}}$$

