

وزارة التعليم العالي والبحث العلمي
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تحليلات هندسية

المرحلة الثالثة
هندسة الحاسبات
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Laplace Transform

The main application of Laplace transformation for us will be solving some differential equations, the Laplace Trans. converts integral and differential equations into algebraic equations. [most diff. eq. with initial values will have a unique solution].

We define the Laplace Transform of a function $f, [0, \infty)$

as : $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$ for $s \in \mathbb{C}$ (complex num.)

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and we use $F(s) = \mathcal{L}\{f(t)\}$

Definition: Given a function $f(t), t \geq 0$, its Laplace Trans.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = \lim_{A \rightarrow \infty} \int_0^A e^{-st} f(t) dt$$

We say the transform converges if the limit exists and diverges if not

* Find the Laplace Trans. using definition

1- $f(t) = 1$ for $t > 0$

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Solution: $F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{1\} = \int_0^{\infty} e^{-st} (1) dt = -\frac{1}{s} \left[e^{-st} \right]_0^{\infty}$
 $= -\frac{1}{s} \left[\frac{1}{e^s} \right]_0^{\infty} = \frac{1}{s}$

①

Find the Laplace trans. using definition
1) $f(t) = 1$

$$\mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} f(t) dt$$

$$= \int_0^{\infty} e^{-st} (1) dt = \int_0^{\infty} e^{-st} dt \quad \leftarrow \begin{matrix} -s \\ -s \end{matrix}$$

$$= -\frac{1}{s} \int_0^{\infty} -s e^{-st} dt = -\frac{1}{s} (e^{-st}) \Big|_0^{\infty} = -\frac{1}{s} \left(\frac{1}{e^{st}} \right) \Big|_0^{\infty}$$

$$= -\frac{1}{s} \left(\frac{1}{e^{\infty}} - \frac{1}{e^0} \right) = -\frac{1}{s} \left(\frac{1}{\infty} - \frac{1}{1} \right) = \frac{1}{s}$$

$$\therefore \mathcal{L}(1) = \frac{1}{s}$$

$$\mathcal{L}\left(\frac{2}{3}\right) = \frac{2}{3s}$$

$$\mathcal{L}(\sqrt{3}) = \frac{\sqrt{3}}{s}$$

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$$2- f(t) = 3$$

Solution:- $F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{3\} = \int_0^{\infty} e^{-st} (3) dt = \frac{3}{s}$

In general If $\boxed{f(t) = a \Rightarrow F(s) = \frac{a}{s}}$

حفظ

$$3- f(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \quad \text{unit step function}$$

Solu: $\mathcal{L}\{u(t)\} = \int_0^{\infty} e^{-st} (1) dt + \int_{-\infty}^0 e^{-st} (0) dt$
 $= \int_0^{\infty} e^{-st} dt \Rightarrow [ex(1)]$

$$4- f(t) = t$$

Solu: $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt = \int_0^{\infty} e^{-st} t dt \quad \text{using } (u)$

$$\int u dv = uv - \int v du$$

$$\int_0^{\infty} e^{-st} t dt = \left[\frac{-t}{s} e^{-st} \right]_0^{\infty} - \int_0^{\infty} \frac{1}{s} e^{-st} dt$$

$$= -\frac{1}{s^2} \left(\frac{1}{s} e^{-st} \right) = \frac{1}{s^2}$$

$$\boxed{\begin{aligned} u &= t \Rightarrow du = dt \\ dv &= e^{-st} dt \Rightarrow v = \frac{1}{s} e^{-st} \end{aligned}}$$

In general

$$\boxed{\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}}$$

 $n \geq 1$

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حفظ

$$f(t) = t^3$$

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Solu: $\mathcal{L}(t^3) = \frac{3!}{s^4} = \frac{6}{s^4}$

6- The Laplace trans. of $(\sin at)$ and $(\cos at)$

Solu: we can find $\mathcal{L}(\sin at)$ and $\mathcal{L}(\cos at)$ using $(u dv)$ twice

or by the Euler's formula, $e^{iat} = \cos at + i \sin at$

$$\therefore \mathcal{L}[e^{iat}] = \int_0^{\infty} e^{-st} e^{iat} dt = \int_0^{\infty} e^{-(s-ia)t} dt = \frac{1}{s-ia}$$

$$\therefore \mathcal{L}[e^{iat}] = \frac{1}{s-ia} \cdot \frac{s+ia}{s+ia} = \frac{s+ia}{s^2+a^2} = \frac{s}{s^2+a^2} + i \frac{a}{s^2+a^2}$$

Comparing the real and the imaginary parts we get

$\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$, $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$ $s > 0$

$$\therefore \mathcal{L}\{\cos 5t\} = \frac{s}{s^2+25} \text{ and } \mathcal{L}\{\sin 3t\} = \frac{3}{s^2+9}$$

H.w find $\mathcal{L}\{\cosh at\}$ and $\mathcal{L}\{\sinh at\}$

(3)

Ex

$$1) \mathcal{L}(t^2) = \frac{2!}{s^3} = \frac{2}{s^3}$$

$$2) \mathcal{L}(t^5) = \frac{5!}{s^6} = \frac{5 \times 4 \times 3 \times 2 \times 1}{s^6} = \frac{120}{s^6}$$

$$3) \mathcal{L}(\cos 2t) = \frac{s}{s^2 + 4}$$

$$4) \mathcal{L}(\cos \sqrt{3}t) = \frac{s}{s^2 + 3}$$

$$5) \mathcal{L}\left(\sin \frac{2}{3}t\right) = \frac{\frac{2}{3}}{s^2 + \frac{4}{9}}$$

$$6) \mathcal{L}(\cosh 5t) = \frac{s}{s^2 - 25}$$

$$7) \mathcal{L}(\sinh 7t) = \frac{7}{s^2 - 49}$$

Find $\mathcal{L}(\sin at)$ and $\mathcal{L}(\cos at)$

Using Euler's formula: $e^{iat} = \cos at + i \sin at$

$$\therefore \mathcal{L}(e^{iat}) = \int_0^{\infty} e^{-st} e^{iat} dt = \int_0^{\infty} e^{-st+iat} dt = \int_0^{\infty} e^{-(s-ia)t} dt$$

$$= \frac{1}{(s-ia)} e^{-(s-ia)t} \Big|_0^{\infty} = \frac{1}{s-ia}$$

$$\therefore \mathcal{L}(e^{iat}) = \frac{1}{s-ia} \times \frac{s+ia}{s+ia} = \frac{s+ia}{s^2+a^2} = \frac{s}{s^2+a^2} + i \frac{a}{s^2+a^2}$$

$$\therefore \mathcal{L} \cos at = \frac{s}{s^2+a^2} \quad / \quad \mathcal{L} \sin at = \frac{a}{s^2+a^2}$$