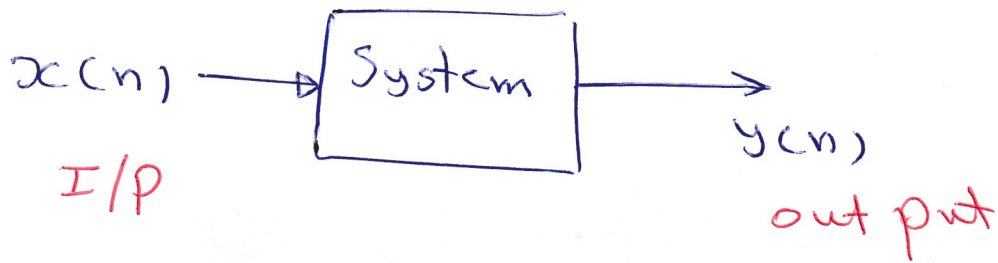


Static and Dynamic System ①

S8D



$$y(n) = x(n-1) \quad \text{past i/p}$$

$$y(n) = x(n) \quad \text{present i/p}$$

$$y(n) = x(n+1) \quad \text{future i/p}$$

Static System [memory less system]

o/p of system depends on present values of input i/p

ex

$$\textcircled{1} \quad y(n) = 2x(n)$$

$$\textcircled{2} \quad y(n) = 5x(n)$$

$$\textcircled{3} \quad y(0) = 2x(0)$$

Dynamic System: o/p of system depends on past or future values of i/p at any instant of time
system with memory

ex

① $y(n) = x(n) + x(n-1)$

present i/p past i/p

② $y(n) = x(n+1) + x(n)$

future i/p present i/p

③ $y(n) = x(2n)$

Time scaling

10) $y(n) = \frac{d}{dn} x(n)$

تفاضل نسبت به n

④ $y(n) = x(-n)$

Time reverse

⑤ $y(n) = \int_{-A}^n x(n) dn$

الانكامل من -A الى n

⑥ $y(n) = x[\sin(n)]$

⑦ $y(n) = x(n-1)$

⑧ $y(n) = x(n+1)$

Time shifting

⑨ $y(n) = x(n^2)$

Time scaling

SSD
③

ex | $y(n) = x(2n)$ Time Scalling

$$n = 0$$

$$y(0) = x(0) \quad \text{present i/p}$$

$$n = -1$$

$$y(-1) = x(-2) \quad \text{past i/p}$$

$$n = 1$$

$$y(1) = x(2) \quad \text{future i/p}$$

$\therefore$ the system is Dynamic

depend of ^{past}, present and future inputs

The system Dynamic if The system Time scalling

ex

$$y(n-2) = x(n-2)$$

past o/p

past i/p

$$n=0 \quad \therefore y(-2) = x(-2)$$

$\therefore -$

past o/p

past i/p

$\therefore$ The system is static

ex

$$y(n+1) = x(n+1)$$

future o/p

future i/p

$$\text{if } n=0 \quad y(1) = x(1)$$

future
o/p

future
o/p

$\therefore$ The system is static

ex

$$y(n) = x(-n)$$

Reverse Time

$$\text{if } n=0$$

$$y(0) = x(0) \quad \text{present i/p}$$

$$n=-1$$

$$y(-1) = x(1)$$

past
o/p

future
i/p

$$n=1$$

$$y(1) = x(-1)$$

future
o/p

past
i/p

The system is Dynamic

ex

$$y(n) = 2x^2(n)$$

$$n=0 \quad y(0) = 2x^2(0)$$

present o/p present i/p

$$y(1) = 2x^2(1)$$

$\therefore$ The system is static

ex

$$y(n) = x(n) + x(n-1)$$

$$n=1$$

$$y(1) = x(1) + x(0)$$

future
i/p

present i/p

$\therefore$ The system is Dynamic

ex

$$y(n) = \sin(n) x(n)$$

$$n=0$$

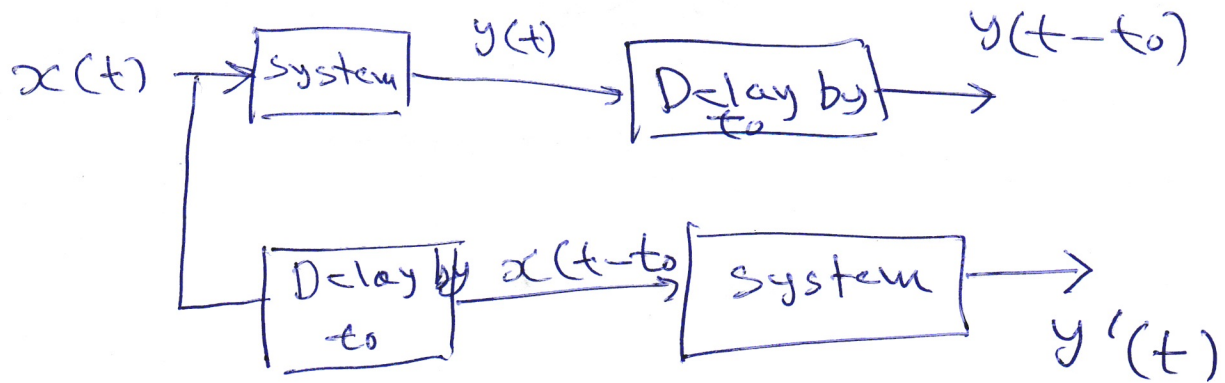
$$y(0) = 0$$

$$n=1$$

$$y(1) = \sin(1) x(1)$$

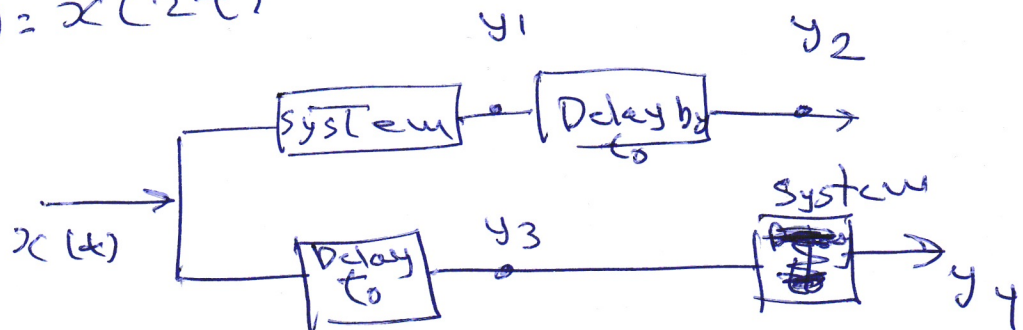
$\therefore$ The system is static

Time-Invariant & Time Variant System



If $y'(t) = y(t-t_0)$ The system is Time invariant
 If $y'(t) \neq y(t-t_0)$ The system is Time Variant

ex | $y(t) = x(2t)$



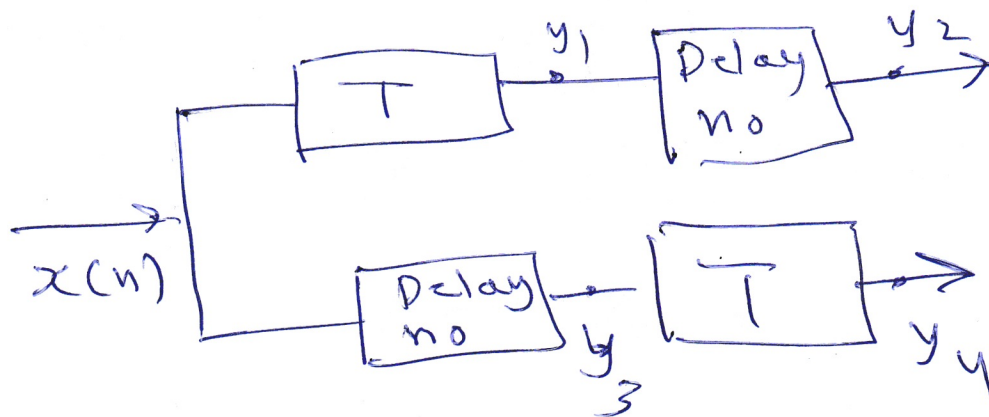
$y_1 = x(2t)$ $y_2 = x(2(t-t_0)) = x(2t-2t_0)$

$y_3 = x(2(t-t_0))$ $y_4 = x(2t-t_0)$

$\therefore y_2 \neq y_4 \therefore$ The system is Time Variant

(2)

Ex 2 $y(n) = 2 + x(n)$



$$y_1(n) = 2 + x(n) \quad y_2(n) = 2 + x(n - n_0)$$

$$y_3(n) = x(n - n_0) \quad y_4(n) = 2 + x(n - n_0)$$

$$y_2(n) = y_4(n)$$

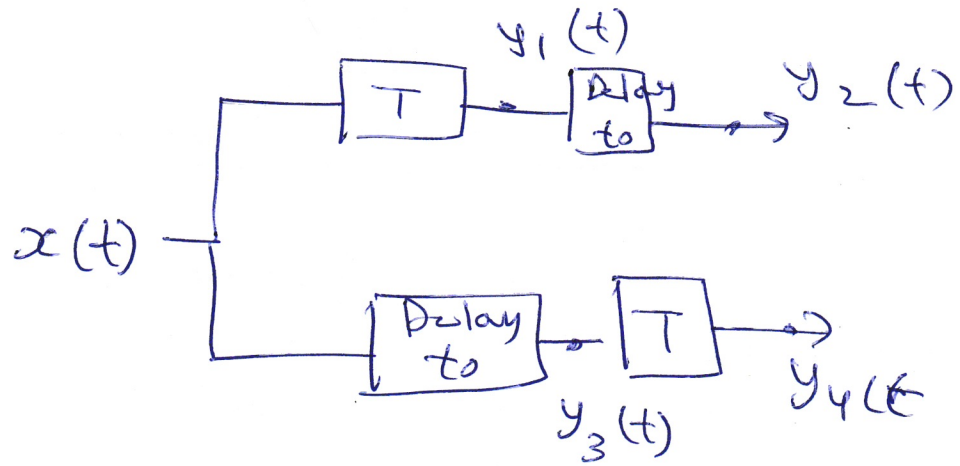
$\therefore$ The system is Time invariant

H-w | check TIV and TV properties of following system

- ① $y(n) = x(\sin n)$
- ② $y(n) = x(n+2)$
- ③ $y(t) = \cos t + x(t)$

TV & TIV
③

ex] $y(t) = x(\cos t)$



$$y_1(t) = x(\cos t)$$

$$y_2(t) = x(\cos(t-t_0))$$

$$y_3(t) = x(t-t_0)$$

$$y_4(t) = x(\cos(t-t_0))$$

$$y_2(t) \neq y_4(t)$$

∴ The system is Time variant [TV]

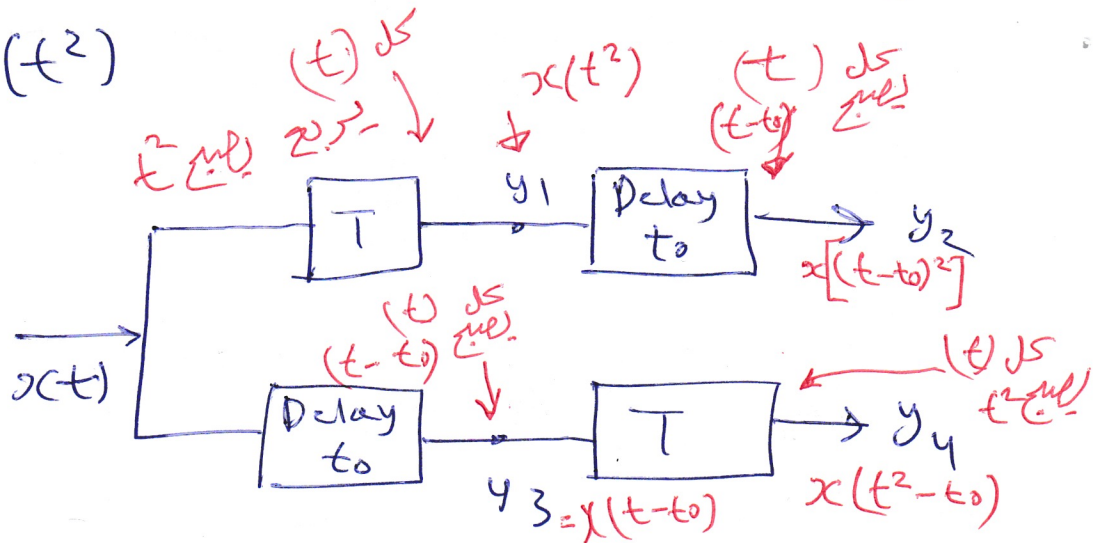
ex] $y(t) = x(Tant)$ as above

The system is Time variant
TV

TV S TV
(4)

ex]

$$y(t) = x(t^2)$$



$$y_1(t) = x(t^2) \quad y_2(t) = x((t-t_0)^2)$$

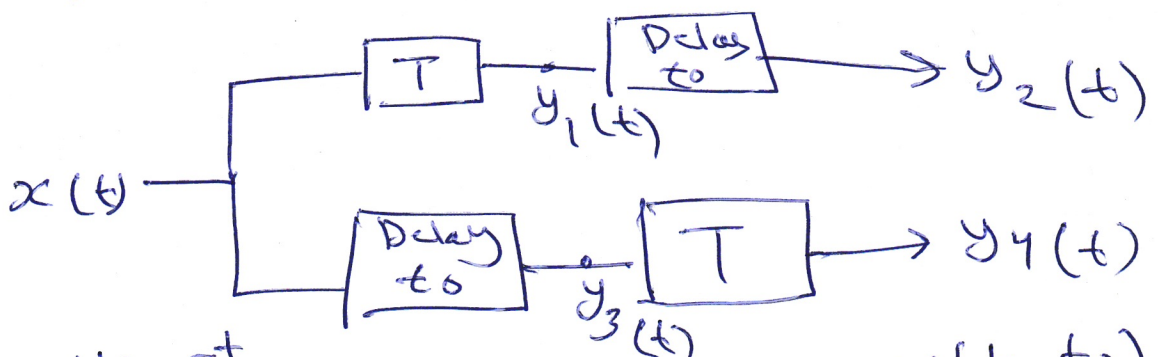
$$y_3(t) = x(t-t_0) \quad y_4(t) = x(t^2-t_0)$$

$$\therefore y_2 \neq y_4$$

The system is T.V.

ex]

$$y(t) = x(t) e^{-t}$$



$$y_1(t) = x(t) e^{-t}$$

$$y_2(t) = x(t-t_0) e^{-(t-t_0)}$$

$$y_3(t) = x(t-t_0)$$

$$y_4(t) = x(t-t_0) e^{-t}$$

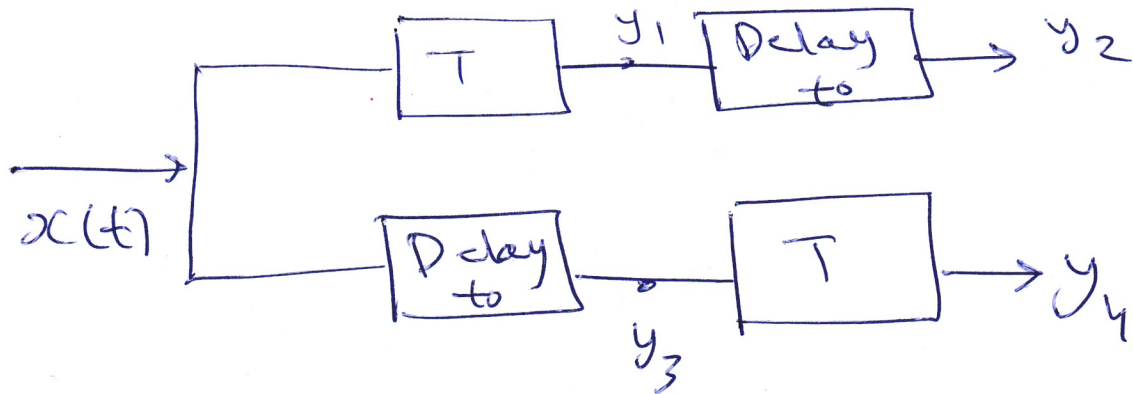
$$\therefore y_3 \neq y_4$$

T.V.

ex

TV & TIV
(5)

$$y(t) = e^{-2k} x(t)$$



$$y_1(t) = e^{-2k} x(t) \quad y_2(t) = e^{-2k} x(t - t_0)$$

$$y_3(t) = x(t - t_0) \quad y_4(t) = e^{-2k} x(t - t_0)$$

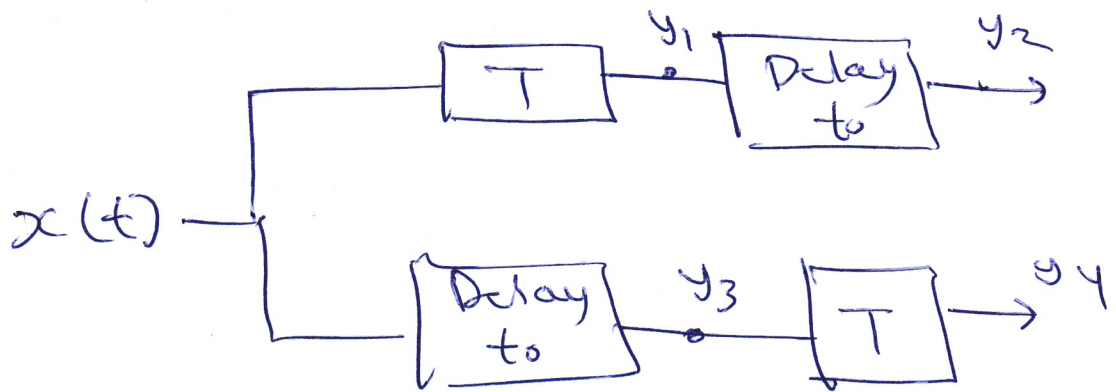
$$y_2 = y_4$$

∴ The system is
Time invariant
(TIV)

ex

TV & TW
⑥

$$y(t) = \tan(t+1) x(t)$$



$$y_1(t) = \tan(t+1) x(t) \quad y_2(t) = \tan(t-t_0+1) x(t-t_0)$$

$$y_3(t) = x(t-t_0) \quad y_4 = \tan(t+1) x(t-t_0)$$

$$\therefore y_2 \neq y_4$$

The system is Time Variant
T.V.

~~Causal & non Causal~~ Causal and non causal system

Causal system : o/p of system
is independent of future value
of i/p

Non - Causal system o/p of system
depends on future values of i/p
at any instant of time

ex

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

for $n = 1$

$$y(1) = x(1) + \frac{1}{x(0)}$$

↑ ↑ ↑
present present past i/p
o/p i/p

The system is Causal

Causal & non Causal

(2)

ex

$$y(t) = 2x(t) + \frac{1}{x^2(t)}$$

if $n=0$

$$\therefore y(0) = 2x(0) + \frac{1}{x^2(0)}$$

present ↑ present : present
o/p i/p i/t

The System is Causal

ex

$$y(n) = x(n) + 2x(n+1)$$

for $n=0$

$$y(0) = x(0) + 2x(1)$$

↑ ↑
present future
i/p i/p

The system is non Causal

Causal & non causal
(3)

ex |

$$y(t) = x(t) + x(t-1) + x(t+1)$$

if $t = 0$

$$y(0) = x(0) + x(-1) + x(1)$$

$\therefore$ non Causal system

Stable and unstable System

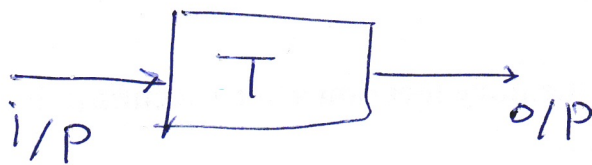
BIBO
①

Bounded i/p. Bounded o/p criteria

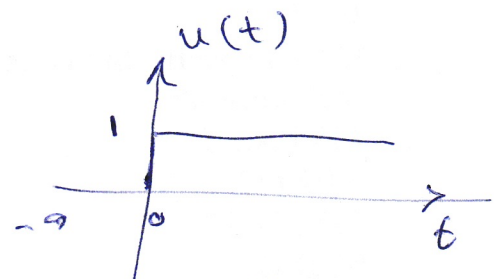
BIBO: for a stable system o/p should be bounded for bounded i/p at each and every instant of time

Bounded signal dc, sint, cost, $u(t)$

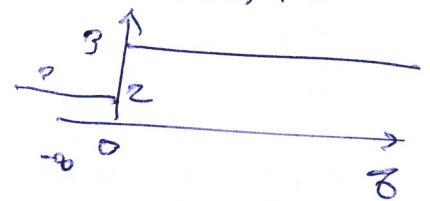
ex 11 $y(t) = x(t) + 2$



bounded i/p as $u(t)$



$\therefore$ o/p $y(t) = u(t) + 2$



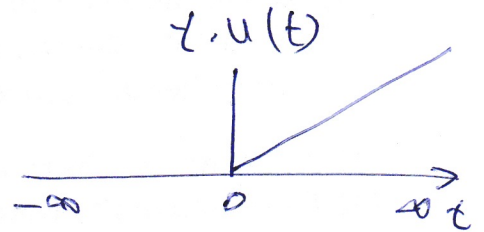
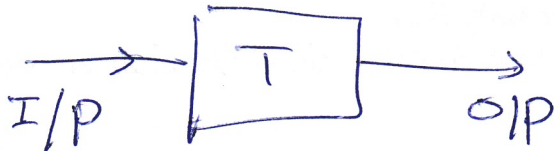
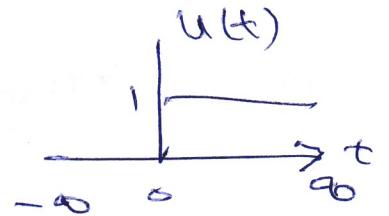
$\therefore$ The system is stable
o/p Bounded

BIBO

exc 2

$$y(t) = t x(t)$$

let the i/p is $u(t)$
bounded i/p



$$y(t) = t \cdot u(t)$$

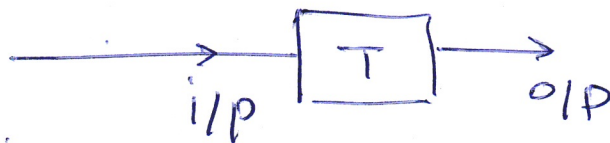
$\therefore$ o/p unbounded

The system is
unstable

exc 3

$$y(t) = \sin[x(t)]$$

$$y(t) = \sin[x(t)]$$



let i/p bounded

$$x(t) = u(t)$$

$$\therefore y(t) = \sin[u(t)] \quad -1 \leq \sin[\] \leq 1$$

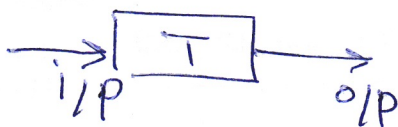
$\therefore$ bounded

BIBO

stable

ex 4)

$$y(t) = \int_{-\infty}^t x(t) dt$$

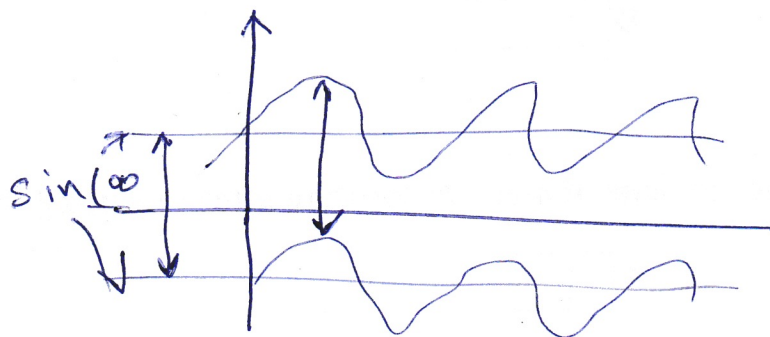


let $x(t) = \cos t$

$$y(t) = \int_{-\infty}^t \cos t dt = \sin(t) \Big|_{-\infty}^t$$

$$y(t) = \sin(t) - \sin(-\infty)$$

$$y(t) = \sin(t) + \underbrace{\sin \infty}_{-1 \leq \sin \infty \leq 1}$$



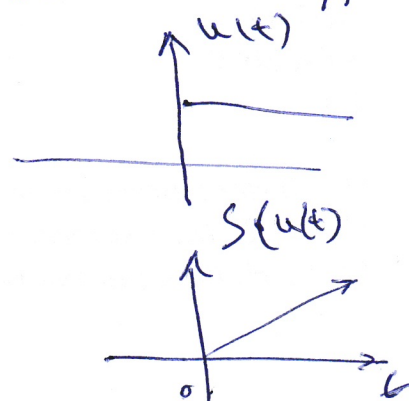
for this bounded o/p

if $x(t) = u(t)$

$$y(t) = \int_{-\infty}^t u(t) dt = \int_0^t 1 dt$$

$$y(t) = t \Big|_0^t = t$$

$\therefore$ o/p unbounded



The System is unstable