Q1 find the value of expression $\left[\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}\right]$

Q3 which of the folloing vectors is perpendicular to the vector $\vec{V} = 2i + 4j$

(a) $\vec{u} = 2i - 4j$ (b) $\vec{w} = -2i + j$ (c) $\vec{x} = i$

Q5 compute the proj $\vec{u}$ where $\vec{u} = 3i + 2j - 3k$ and $\vec{v} = i + j + 3k$

Q7 find the Eigen value and Eigen vectors of

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Q9 for which value of a and b the matrix

$$B = \begin{bmatrix} 3 & 1+i & 2+6i \\ a & -1 & 2-4i \\ 2-6i & b & 1 \end{bmatrix}$$ is Hermitian ?

Q11 Given the vectors $\vec{a} = 2i - k$ and $\vec{b} = 2i + 3j$ find the unite vector in direction of $\vec{a} \times \vec{b}$?

Q13 find the value of the exeprssion $Z = (2+2i)^5$ using Polar form ?

Q15 Simplify and find the value of i^{4n-5}

2) The value of the expression

$$\left[\frac{2+3i}{3-2i}\right]^6$$

4/ Find the three roots of $Z=-16$ in the form $x+iy$

Find the square roots of $Z=-1-i$ in the form $x+iy$

6/ Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}3 & 0 & 0\\ 0 & 4 & 4\\ 0 & 2 & 5\end{bmatrix}$

8/ solve the system linear equations by Gauss elimination:

$$x_1 + x_2 + 2x_3 = 1$$

$$-x_1 - x_2 + x_3 = 5$$

$$x_1 - x_2 + 3x_3 = 3$$

10/ Given the vectors $\vec{A} = -2i - 5j + 2k$ and $\vec{B} = -5i - 2j - 3k$, then find $\|\vec{A} \times \vec{B}\|$

12/ Is the matrix $B = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ a ~~unitary~~ unitary matrix ?

14/ If $z_1 = 3 + 4i$ and $z_2 = 1 + 2i$ find $\overline{z_1 \cdot z_2}$

$= \overline{z_1} \cdot \overline{z_2}$

16/ Evaluate the double integral

$$\iint_R \frac{\sin x}{x} \, dA$$ where R is the triangle in the xy-Plane bounded by the x-axis the line $y = x$ and $x = 1$

Q1) find the value of expression

$$\left[\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+1}\right]$$

Sol)

$$= \left[\frac{1+i}{1-i}\cdot\frac{1+i}{1+i} - (2+2i+4i+4i^2) + \frac{3-i}{1+i}\cdot\frac{1-i}{1-i}\right]$$

$\because i^2 = -1$

$$= \left[\frac{(1+i)(1+i)}{1+1} - (2+6i-4) + \frac{(3-i)(1-i)}{1+1}\right]$$

$$= \left[\frac{(1+i+i+i^2)}{2} - (-2+6i) + \frac{(3-3i-i+i^2)}{2}\right]$$

$$= \left[\frac{1+2i-1}{2} - (-2+6i) + \frac{3-4i-1}{2}\right]$$

$$= \left[\frac{2i}{2} - (-2+6i) + \frac{2-4i}{2}\right]$$

$$= \left[i - (-2+6i) + \frac{2}{2} - \frac{4i}{2}\right]$$

$$= [i+2-6i+1-2i]$$

$$= 3-7i$$

Q2] Find the value of the expression

$$\left[\frac{2+3i}{3-2i}\right]^6$$

Sol]

$$= \left[\frac{2+3i}{3-2i} \cdot \frac{3+2i}{3+2i}\right]^6$$

$$= \left[\frac{(2+3i)\cdot(3+2i)}{9+4}\right]^6$$

$$= \left[\frac{6+4i+9i+6i^2}{13}\right]^6$$

$\because i^2 = -1$

$$= \left[\frac{6+13i-6}{13}\right]^6$$

$$= \left[\frac{13i}{13}\right]^6 \cdot 2\left[i\right]^6 \qquad = i^6$$

$i^6 = -1$.

Q3) which of the following vectors is perpendicular to the vector $\vec{v} = 2i + 4j$

(a) $\vec{u} = 2i - 4j$ (b) $\vec{w} = -2i + j$ (c) $\vec{x} = i$

sol

$$\vec{a} \cdot \vec{b} = |a| \cdot |b| \cdot \cos(\theta)$$

① $\vec{u} = 2i - 4j$

$$\vec{v} \cdot \vec{u} = (2)(2) + (4)(-4)$$
$$= 4 - 16 = -12$$

② $\vec{w} = -2i + j$

$$\vec{v} \cdot \vec{w} = (2)(-2) + (4)(1)$$
$$= -4 + 4 = 0$$

③ $\vec{x} = i$

$$\vec{v} \cdot \vec{x} = (2)(1) + (4)(0)$$
$$= 2 + 0 = 2$$

$\therefore \vec{w}$ is prependicular to $\vec{v}$.

Q4) A) Find the three roots of $z = -16$ in the form $x+iy$.

B) Find the square roots of $z = -1-i$ in the form $x+iy$.

Sol//

A) $z = -16 \Rightarrow z = -16+0i$

$r = \sqrt{x^2+y^2} \Rightarrow r = \sqrt{(-16)^2+(0)^2}$

$r = \sqrt{256+0} = 16$

$\tan^{-1}\theta = \frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{y}{x} \Rightarrow \theta = \tan^{-1}\frac{0}{-16} = (\text{undefined})$

$\theta = \pi - \theta \Rightarrow \theta = \pi - (\text{undefined})$

$\theta = \pi$

		sin		cos		tan	
$\pi-\theta$	θ	+	+	−	+	−	+
$\pi+\theta$	$2\pi-\theta$	−	−	−	+	+	−

$z = r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2\pi k}{n}\right)\right]$

$k = 0,1,2$

$n = 3$

$k = 0$

$z_1 = (16)^{\frac{1}{3}}\left[\cos\left(\frac{\pi+2.0.\pi}{3}\right)+i\sin\left(\frac{\pi 2.0.\pi}{3}\right)\right]$

$k = 1$

$z_2 = (16)^{\frac{1}{3}}\left[\cos\left(\frac{\pi+2.1.\pi}{3}\right)+i\sin\left(\frac{\pi+2.1.\pi}{3}\right)\right]$

$k = 2$

$z_3 = (16)^{\frac{1}{3}}\left[\cos\left(\frac{\pi+2.2.\pi}{3}\right)+i\sin\left(\frac{\pi+2.2.\pi}{3}\right)\right]$

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B) $z = -1 - i$

$z = 2(\cos 60 + i \sin 60)$

$= 2^{\frac{1}{2}}\left[\cos\left(\frac{60}{2} + k \times \frac{360}{2}\right) + i \sin\left(\frac{60}{2} + k \times \frac{360}{2}\right)\right]$

$= \sqrt{2}\left[\cos(30 + k \times 180) + i \sin(30 + k \times 180)\right]$

$k = 0, 1$

$k = 0$

$= \sqrt{2}(\cos 30 + i \sin 30)$

$k = 1$

$= \sqrt{2}(\cos 210 + i \sin 210)$

Q5) compute the $Proj_{\vec{u}}\vec{v}$ where $\vec{u} = 3i+2j-3k$ and $\vec{V} = i+j+3k$

Sol)

$$Proj_u V = \frac{u.v}{|u|^2} \cdot u$$

$$u.v = (3i+2j-3k).(i+j+3k)$$
$$= (3.1)+(2.1)+(-3.3)$$
$$= 3+2-9 = -4$$

$$|u|^2 = (3i+2j-3k).(3i+2j-3k)$$
$$= (3.3)+(2.2)+(-3.-3)$$
$$= 9+4+9 = 22$$

$$Proj_u V = \frac{-4}{22} \cdot (3i+2j-3k)$$
$$= -\frac{2}{11}(3i+2j-3k)$$
$$= -\frac{6}{11}i - \frac{4}{11}j + \frac{6}{11}k$$

Q6 Find the Eigen values and Eigen vectors of $A=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 4 \\ 0 & 2 & 5 \end{bmatrix}$

Sol

1-Eigen values

$$A-\lambda I=\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 4-\lambda & 4 \\ 0 & 2 & 5-\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=(3-\lambda)[(4-\lambda)(5-\lambda)-(4)(2)]$$

$$(3-\lambda)[(4-\lambda)(5-\lambda)-8]=0$$

$$(3-\lambda)(\lambda^2-9\lambda+12)=0$$

$$(3-\lambda)(\lambda-3)(\lambda-4)=0$$

$$\lambda_1=3,\ \lambda_2=4,\ \lambda_3=5$$

2-Eigen vectors:

for $\lambda_1=3$

$$A-3I=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

$$V_1=\begin{bmatrix} 0 \\ -4 \\ 2 \end{bmatrix}$$

$\lambda_2=4$

$$A-4I=\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 2 & 1 \end{bmatrix}$$

$$V_2=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda_3=5$

$$A-5I=\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$

$$V_3=\begin{bmatrix} 0 \\ -4 \\ 1 \end{bmatrix}$$

Q 7] Find the Eigen value and Eigen vectors of $A=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Sol] 1- Eigenvalue

$$A-\lambda I=\begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix}$$

$$\det(A-\lambda I)=(1-\lambda)(1-\lambda)-(-1)(1)$$

$$\lambda^2-2\lambda+2=0$$

$$\lambda=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=1, b=-2, c=2$$

$$\lambda=\frac{2\pm\sqrt{4-8}}{2}\Rightarrow\frac{2\pm\sqrt{-4}}{2}\Rightarrow\frac{2\pm 2i}{2}=1\pm i$$

$$\lambda_1=1+i\ ,\ \lambda_2=1-i$$

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2- Eigenvectors

for $\lambda_1 = 1+i$

slove $(A-(1+i)I)V=0$:

$$\begin{bmatrix} 1-(1+i) & -1 \\ 1 & 1-(1+i) \end{bmatrix} V = 0$$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} V = 0$$

① $-iV_1 - V_2 = 0$ and ② $V_1 - iV_2 = 0$

$-iV_1 - V_2 = 0 \Rightarrow V_1 = -iV_2$

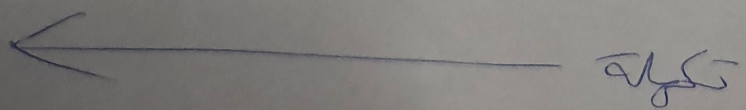
put in second equation

$-iV_2 - iV_2 = 0$

$-iV_2 - iV_2 = 0$

$-2iV_2 = 0 \Rightarrow V_2 = 0$

$V_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$



for $\lambda_2 = 1 - i$:

Solve $(A - (1-i)I)V = 0$

$$\begin{bmatrix} 1-(1-i) & -1 \\ 1 & 1-(1-i) \end{bmatrix} V = 0$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} V = 0$$

① $iv_1 - v_2 = 0$ and ② $v_1 + iv_2 = 0$

$iv_1 - v_2 = 0 \Rightarrow v_1 = v_2 i$

Put in equation second

$v_2 i + iv_2 = 0 \Rightarrow 2iv_2 = 0 \Rightarrow v_2 = 0$

$$v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Q8) solve the system linear equation by Gauss elimination:

$$X_1 + X_2 + 2X_3 = 1$$
$$-X_1 - X_2 + X_3 = 5$$
$$X_1 - X_2 + 3X_3 = 3$$

Sol)

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ -1 & -1 & 1 & 5 \\ 1 & -1 & 3 & 3 \end{array}\right]$$

$\therefore R_2 + 1R_1 \rightarrow R_2$

$\therefore R_3 - 1R_1 \rightarrow R_3$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 0 & 3 & 6 \\ 0 & -2 & 1 & 2 \end{array}\right]$$

$\therefore R_2 \longleftrightarrow R_3$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$\therefore R_2 / -2 \longrightarrow R_2$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -0.5 & -1 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$\therefore R_1 - 1R_2 \longrightarrow R_1$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2.5 & 2 \\ 0 & 1 & -0.5 & -1 \\ 0 & 0 & 3 & 6 \end{array}\right]$$

$\therefore R_3 / 3 \longrightarrow R_3$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2.5 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore R_1 - 2.5R_3 \longrightarrow R_1$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$X = -3, X_2 = 0, X_3 = 2$

Q9) for which value of a and b the matrix $B \begin{bmatrix} 3 & 1+i & 2+6i \\ a & -1 & 2-4i \\ 2-6i & b & 1 \end{bmatrix}$ is Hermition?

Sol)

$$B^T = \begin{bmatrix} 3 & a & 2-6i \\ 1+i & -1 & b \\ 2+6i & 2-4i & 1 \end{bmatrix}$$

$$\overline{B^T} = \begin{bmatrix} 3 & a & 2+6i \\ 1-i & -1 & b \\ 2-6i & 2+4i & 1 \end{bmatrix}$$

the matrix B is not Hermition

Q10 Given The vectors $\vec{A} = -2i - 5j + 2k$ and $\vec{B} = -5i - 2j - 3k$, then find $\|\vec{A} \times \vec{B}\|_2$

Sol

$$A \times B = \begin{bmatrix} i & j & k \\ -2 & -5 & 2 \\ -5 & -2 & -3 \end{bmatrix}$$

$$A \times B = (i)\begin{vmatrix} -5 & 2 \\ -2 & -3 \end{vmatrix} - (j)\begin{vmatrix} -2 & 2 \\ -5 & -3 \end{vmatrix} + (k)\begin{vmatrix} -2 & -5 \\ -5 & -2 \end{vmatrix}$$

$$A \times B = (i)(-5(-3) - 2(-2)) - (j)(-2(-3) - 2(-5)) + (k)(-2(-2) - (-5)(-5))$$

$$A \times B = (i)(15 + 4) - (j)(6 + 10) + (k)(4 - 25)$$

$$A \times B = (19i) - (16j) - (21k)$$

$$\|A \times B\| = \sqrt{(19)^2 + (-16)^2 + (-21)^2}$$

$$\|A \times B\| = \sqrt{361 + 256 + 441}$$

$$\|A \times B\| = \sqrt{1058} = 32.55$$

Q11/ Given the vectors $\vec{a} = 2i - k$ and $b = 2i + 3j$ find the unite vector in direction $\vec{a} \times \vec{b}$?

Sol/

$$\vec{a} = 2i + 0i + k$$
$$\vec{b} = 2i + 3i + 0k$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} i & j & k \\ 2 & 0 & -1 \\ 2 & 3 & 0 \end{bmatrix}$$

$$\vec{a} \times \vec{b} = (0 \times 0) - (1 \times 3)i - (2 \times 0) - (1 \times 2)j + (2 \times 5) - (0 \times 2)k$$

$$\vec{a} \times \vec{b} = 3i + 2i + 6k$$

$$\overrightarrow{a \times b} = \sqrt{(3)^2 + (2)^2 + (6)^2}$$

$$\overrightarrow{a \times b} = \sqrt{9 + 4 + 36}$$

$$\overrightarrow{a \times b} = \sqrt{49} = 7$$

$$u = \frac{V}{|V|} = \frac{3i + 2i + 6k}{7}$$

$$u = \frac{3}{7}i + \frac{2}{7}j + \frac{6}{7}k$$

Q12] is the matrix $B=\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$ a unitary matrix?

Sol]

$$B'=\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}' \Rightarrow B'=\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$$

$$B'B=\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

the B matrix is not a unitary matrix.

Q13] Find the value of the expression

$z=(2+2i)^5$ using polar form?

sol)

$z^n = r^n[\cos n\theta + i\sin n\theta]$

$r=\sqrt{a^2+b^2}$, $z=a+bi$

$r=\sqrt{(2)^2+(2)^2} \Rightarrow r=\sqrt{4+4} \Rightarrow r=\sqrt{8}$

$r=\sqrt{2\cdot4} \Rightarrow r=2\sqrt{2}$

$\cos Q = \frac{x}{r} \Rightarrow \frac{2}{2\sqrt{2}} = \frac{1}{2}$

$\sin Q = \frac{y}{r} \Rightarrow \frac{2}{2\sqrt{2}} = \frac{1}{2}$

$Q = 45^\circ$ or $\frac{\pi}{4}$

$z^5=(2\sqrt{2})^5\left[\cos 5\frac{\pi}{4} + i\sin 5\frac{\pi}{4}\right]$

$z^5=(2\sqrt{2})^5\left[\cos \frac{5\pi}{4} + i\sin \frac{5\pi}{4}\right]$

Q14) If $z_1 = 3+4i$ and $z_2 = 1+2i$ find

$$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

A) $\overline{z_1 \cdot z_2}$

$= \overline{(3+4i)-(1+2i)}$

$= \overline{3+6i+4i+8i^2}$

$\because i^2 = -1$

$= \overline{3+6i+4i-8}$

$= \overline{-5+6i+4i}$

$= \overline{-5+10i}$

$= 5-10i$

B) $\overline{z_1} \cdot \overline{z_2}$

$= \overline{(3+4i)} - \overline{(1+2i)}$

$= 3+6i+4i+8i^2$

$\because i^2 = -1$

$= 3+6i+4i-8$

$= -5+6i+4i$

$= -5+10i$

$= 5-10i$

$\therefore \overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

Q 15] Simplify and find the value of i^{4n-5}

Sol]

$= i^{4n} \cdot i^{-5}$

$= (i^4)^n \cdot \frac{1}{i^5}$

$= -i^4 = 1$

$= (1)^n \cdot \frac{1}{i^5}$

$= 1 \cdot \frac{1}{i^5}$

$= \frac{1}{i^5}$

$= \frac{1}{i^3 \cdot i^2} \Rightarrow \frac{1}{(-i)\cdot(-1)}$

$= \frac{1}{i}$

$= \frac{1}{i} \cdot \frac{-i}{-i}$

$= \frac{-i}{-i^2}$

$= \frac{-i}{-(-1)}$

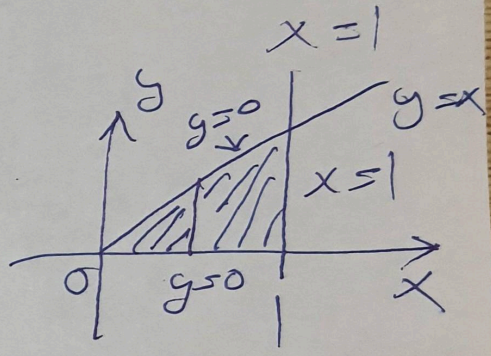
$= \frac{-i}{1}$

$= -i$

Q16) Evaluate the double intergral

$\iint_R \frac{\sin x}{x} dA$ where R is the triangle in the x y-plane boundred by the x-axis the line $y = x$ and $x = y$?

Sol)



1- for x: $0 \leq x \leq 1$

2- for y: $0 \leq y \leq x$

3- for x and y simultaneously $2y \leq x \leq 1$

$$\iint_R \frac{\sin(x)}{x} dA$$

$$= \int_0^1 \int_0^x \frac{\sin(x)}{x} dy\, dx$$

y from 0 to x

$$= \int_0^x \frac{\sin(x)}{x} dy \Rightarrow y \frac{\sin(x)}{x} \Big|_0^x \Rightarrow x \frac{\sin(x)}{x} - 0$$

x from 0 to 1

$$= \sin(x) \Rightarrow \int_0^1 \sin(x) dx = -\cos(x) \Big|_0^1$$

$$= -\cos(1) + \cos(0) = -\cos(1) + 1$$