

Select the Correct Answer from as! ①

1. From the end solution the value of

$$1 + i^2 + i^4 + i^6 + \dots + i^{2n} \text{ is}$$

- a. positive
- ✓ b. Cannot be evaluated.
- c. Negative
- d. 0

Solution :: $n = 0, 1, 2, 3, 4$

$$1 + i^2 + i^4 + i^6 + \dots + i^{2n} = 1 - 1 + 1 - 1 + \dots (-1)^n$$

This cannot be evaluated unless the value of n is known.

~~✗ ✗~~

2. If the complex number of $Z = x + iy$

~~✗~~ Satisfies the condition of the $|Z+1|=1$,

Then Z lies on:

- ✓ a. Circle with center $(-1, 0)$ and radius 1.
- b. Circle with center $(1, 0)$ and radius 1.
- c. X-axis.
- d. Y-axis.

Solution :: $Z = x + iy$ and $|Z+1|=1$

$$\therefore |x + iy + 1| = 1 \Rightarrow |(x+1) + iy| = 1$$

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$$\sqrt{(x+1)^2 + y^2} = 1 \Rightarrow (x+1)^2 + y^2 = 1$$

This is equation of a circle with center $(-1, 0)$ and radius 1

(2)

3. The simplified value of the $\frac{(1-i)^3}{(1-i^3)}$ is

a. 1

b. -i

c. -2

d. 2i

Solution: $\frac{(1-i)^3}{(1-i^3)}$ = $\frac{(1-i)^3}{(1-i)(1+i+i^2)}$

$\Rightarrow \frac{(1-i)^2}{(1+i+i^2)} = \frac{(1-i)^2}{(1+i-1)} = \frac{(1-i)^2}{i} =$

$\frac{1-2i+i^2}{i} = \frac{(1-2i-1)}{i} = \frac{-2i}{i} = \boxed{-2}$

4. Find the values of x and y from the $(X+3) + i(Y-2) = 5 + 2i$:

a. $X=8$ and $Y=4$.b. $X=2$ and $Y=0$.c. $X=8$ and $Y=0$.d. $X=2$ and $Y=4$.

Solution: Real parts of both are equal and imaginary parts of both are equal.

$(X+3) = 5 \Rightarrow X = 5-3 \Rightarrow \boxed{X=2}$

$(Y-2) = 2 \Rightarrow \boxed{Y=4}$

$\boxed{X=2 \text{ and } Y=4}$

5. Find the sum of two complex number. (3)
If $Z_1 = 2+3i$ and $Z_2 = 5+2i$?

- a. $7-5i$
- b. $4+8i$
- c. $3-i$
- ✓ d. $7+5i$

Solution:- The sum $(2+5) + (3+2)i = \boxed{7+5i}$

6. what will be the value of K , so that the roots of the equation are $x^2 + \underline{2K}x + 9$ are imaginary?

- ✓ a. $-3 < K < 3$
- b. $3 < K < -3$
- c. $-5 < K < 3$
- d. $-5 < K < 5$

Solution:- Roots are imaginary

$$\therefore b^2 - 4ac < 0$$

$$(2K)^2 - 4(1)(9) < 0$$

$$4K^2 - 36 < 0 \quad (\div 4)$$

$$K^2 - 9 < 0$$

$$K^2 < 9$$

$$\therefore K < \pm 3 \quad \rightarrow$$

$$-3 < K < 3$$

④

7. Find $Z_1 Z_2$. If $Z_1 = 1 + 2i$ and $Z_2 = 2 + 3i$

a. $2 + 6i$

✓ b. $-4 + 7i$

c. $8 + 7i$

d. $-4 + 0i$

Solution:- $Z_1 Z_2 = (1 + 2i)(2 + 3i)$
 $= 2 + 3i + 4i - 6$
 $Z_1 Z_2 = -4 + 7i$

8. If $\begin{bmatrix} 2p+q & p-2q \\ 5r-s & 4r+3s \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$,

then the value of $p + q - r + 2s$ is equal:

a. 10

b. 4

✓ c. 8

d. -8

Solution:-

$2p + q = 4$ — (1) (x2)

$p - 2q = -3$ — (2)

$5r - s = 11$ — (3)

$4r + 3s = 24$ — (4)

eq. (1) x2 + eq. (2), we find;

~~$4p + q = 8$~~ $4p + 2q = 8$ $\Rightarrow 5p = 5$
 $+ \quad p - 2q = -3$ $\therefore p = 1$

⑤

From eqn. ① we put $p=1$ we get: (1) $\Rightarrow$ $p=1$ في المعادله ①

$$2p + q = 4 \Rightarrow 2(1) + q = 4 \Rightarrow q = 4 - 2$$

$$\therefore \boxed{q=2}$$

- eqn ③ $\times 3$ + eqn ④, we find

$$5r - s = 11 \quad (\times 3)$$

$$+ \begin{array}{r} 15r - 3s = 33 \\ 4r + 3s = 24 \end{array}$$

$$\hline 19r = 57$$

$$\therefore r = \frac{57}{19} \Rightarrow \boxed{r=3}$$

substituting $r=3$ in eqn ③ we find:

نقوم بوضع $r=3$ في المعادله ③

$$5r - s = 11$$

$$5r - s = 11 \Rightarrow 5(3) - s = 11 \Rightarrow$$

$$s = 15 - 11 \Rightarrow \boxed{s=4}$$

Now;

$$p + q - r + 2s = 1 + 2 - 3 + 2(4) \\ = 1 + 2 - 3 + 8$$

$$\therefore \boxed{p + q - r + 2s = +8}$$

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9. The matrix of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is named as:

- ✓ a. symmetric matrix
- b. identity matrix
- c. skew symmetric matrix
- d. ~~None~~ None of these above.

Solution: From matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

Find Transpose of matrix A

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = A$$

$$\therefore A^T = A$$

$\therefore$ A is a symmetric matrix.

10. If the determine $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$,
then the value of x is find from;

- a. 3
- ✓ b. ± 6
- c. ± 3
- d. 6

(7)

Solution :: $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$$(2x)(x) - (5)(8) = (6)(3) - (-2)(7)$$

$$2x^2 - 40 = 18 + 14$$

$$2x^2 = 18 + 14 + 40$$

$$2x^2 = 72 \xrightarrow{(\div 2)} x^2 = 36$$

$$\boxed{\therefore x = \pm 6}$$

11. The matrix $A = \begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$ is named as:

- a. Symmetric.
- b. skew-symmetric.
- ✓ c. skew-Hermitian.
- d. Hermitian.

Solution ::

From ~~the~~ $A = \begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$

Transpose A is

$$A^T = \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \quad \boxed{\therefore A^T \neq A}$$

$\therefore$ matrix is not symmetric

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$$-A = \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix}$$

$$A^T \neq -A$$

$\therefore$ matrix is not skew-symmetric.

Now Conjugate of matrix A

$$\begin{aligned} \bar{A} &= \begin{bmatrix} 0 & \overline{-4+i} \\ \overline{4+i} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -4-i \\ 4-i & 0 \end{bmatrix} \\ &= - \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \end{aligned}$$

$$\therefore \bar{A} = -A^T$$

$\therefore$ matrix is skew-hermitian matrix

Hermitian Matrix $\boxed{\bar{A}^T = A}$

$$A^T \neq \bar{A}$$

matrix is not Hermitian matrix.

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12. Find the $Z = 1 + i$ in the polar form, then solution is;

a. $Z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$

b. $Z = \sqrt{2} (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$

c. $Z = \sqrt{2} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

d. None of these above.

Solution :-

$Z = 1 + i$ — (1)

From polar form $Z = r \cos \theta + i r \sin \theta$ — (2)

Comparing (1) and (2) we find:

$1 = r \cos \theta$ and $1 = r \sin \theta$

by squaring and adding, we find

* $1 = r^2 \cos^2 \theta$, $1 = r^2 \sin^2 \theta \Rightarrow \frac{1 = r^2 \cos^2 \theta}{1 = r^2 \sin^2 \theta}$
 $r^2 \cos^2 \theta + r^2 \sin^2 \theta = 2$
 $2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$

$r^2 = 2 \Rightarrow \boxed{r = \sqrt{2}}$

There for $\cos \theta = \frac{1}{\sqrt{2}}$; $\sin \theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \frac{\pi}{4}$

ب. $\theta = \frac{\pi}{4}$ و $r = 2$
 (2) $\hat{r}$ $\hat{\theta}$ $\hat{\phi}$

$\therefore Z = \sqrt{2} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

(10)

13. The value of the double integral $\int_0^\pi \int_0^x \frac{\sin x}{x} dx dy$ is equal:

- a. 1
- ✓ b. 2
- c. -2
- d. -1

Solution:-

$$I = \int_0^\pi \int_0^x \frac{\sin x}{x} dx dy = \int_0^\pi \frac{\sin x}{x} dx \int_0^x dy$$

$$\Rightarrow \int_0^\pi \frac{\sin x}{x} dx [y]_0^x \Rightarrow \int_0^\pi \frac{\sin x}{x} dx (x-0)$$

$$\Rightarrow \int_0^\pi \frac{x \sin x}{x} dx \Rightarrow \int_0^\pi \sin x dx$$

$$\therefore I = [-\cos x]_0^\pi = [1 - (-1)]$$

$$\therefore I = [1+1]$$

$$\boxed{\therefore I = 2}$$

11) 14. which of the following is a unit vector?

✓ a. $\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$

b. $\hat{i} + \frac{\hat{j}}{\sqrt{2}}$

c. $\hat{i} + \hat{j}$

d. All of the above.

Solution:

$$\hat{i} + \hat{j} = \sqrt{(1)^2 + (1)^2} = \boxed{\sqrt{2}}$$

$$\hat{i} + \frac{\hat{j}}{\sqrt{2}} = \sqrt{(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{\frac{\sqrt{3}}{\sqrt{2}}}$$

$$\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}} = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \boxed{1}$$

∴ $\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$ is the unit vector.