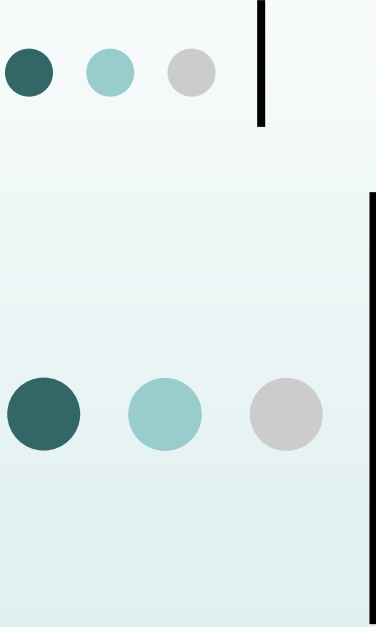




Lecture (1)

Complex Numbers

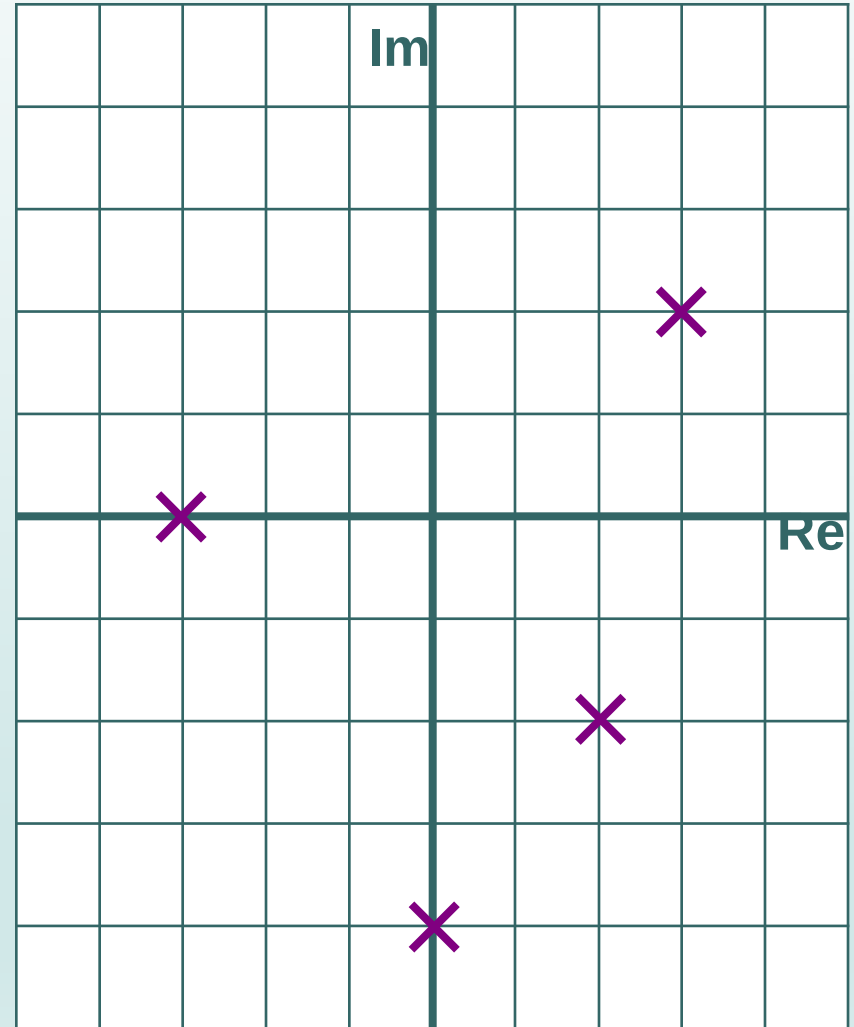
Prof. Dr. Salah S. Abed AlKareem

Argand Diagram

Complex numbers can be shown Geometrically on an Argand diagram

The real part of the number is represented on the x-axis and the imaginary part on the y.

- -3
- -4i
- $3 + 2i$
- $2 - 2i$



Modulus of a complex number

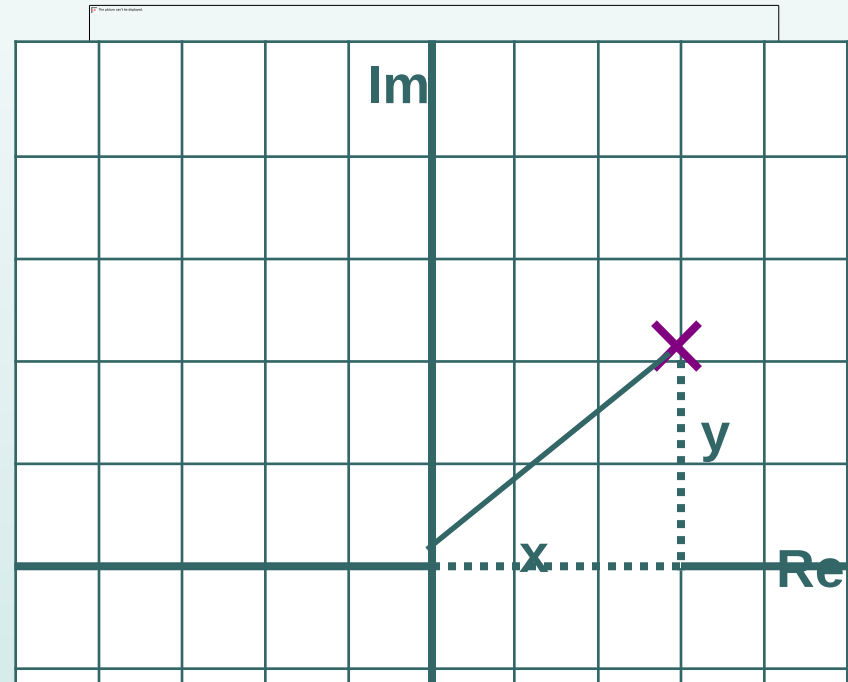
○ A complex number can be represented by the position vector.

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

○ The Modulus of a complex number is the distance from the origin to the point.

Can you generalise this?

$$|z| = \sqrt{x^2 + y^2}$$



How many complex numbers in the form $a + bi$ can you find **with integer values of a and b** that share the same modulus as the number above.

Could you mark all of the points?

What familiar shape would you draw?(more of LOCI later!)

Modulus questions

Find

a) $|3 + 4i| = 5$

b) $|5 - 12i| = 13$

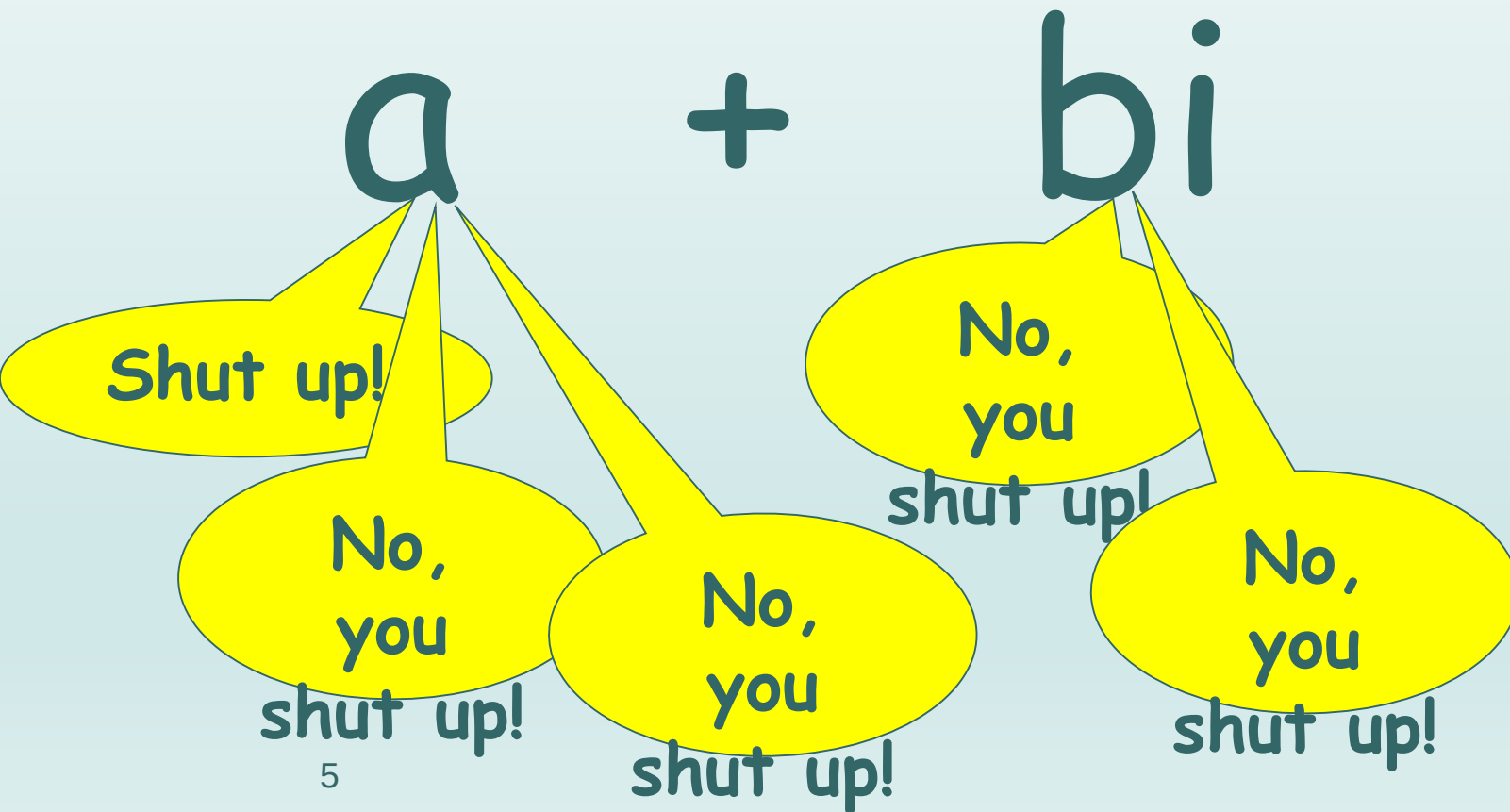
c) $|6 - 8i| = 10$

d) $|-24 - 10i| = 26$

Find the distance between the first two complex numbers above.

It may help to sketch a diagram

The argument of a complex number



The argument of a complex number

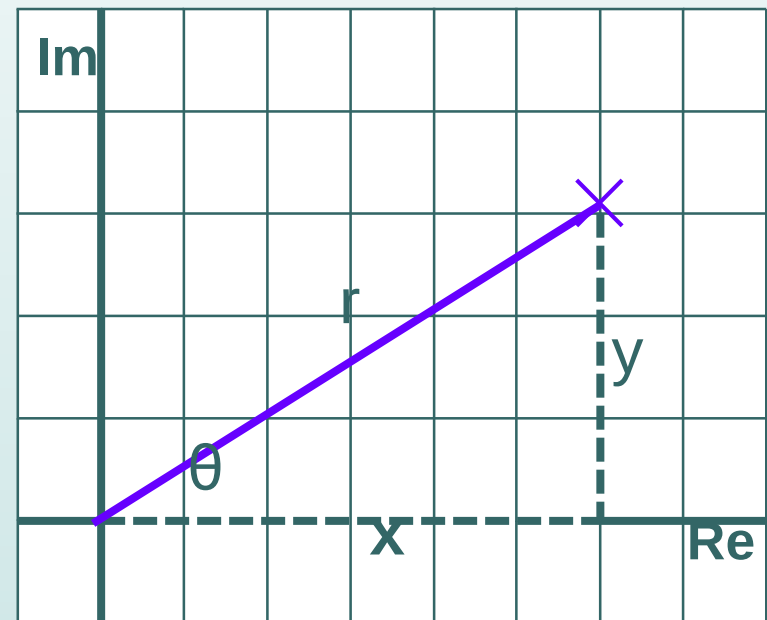
The argument of a complex number is the angle the line makes with the positive x-axis.

Can you generalise this?

$$\arg z = \theta$$

$$\theta = \tan^{-1}(y / x)$$

$$-\pi \leq \theta \leq \pi$$



It is really important that you sketch a diagram **before** working out the argument!!

The argument of a complex number

○ Calculate the modulus and argument of the following complex numbers. (Hint, it helps to draw a diagram)

1) $3 + 4i$ $|z| = \sqrt{3^2 + 4^2} = 5$

$$\begin{aligned}\arg z &= \tan^{-1}(4/3) \\ &= 0.927\end{aligned}$$

2) $5 - 5i$ $|z| = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

$$\begin{aligned}\arg z &= \tan^{-1}(5/-5) \\ &= -\pi/4\end{aligned}$$

3) $-2\sqrt{3} + 2i$ $|z| = \sqrt{(2\sqrt{3})^2 + 2^2} = 4$

$$\begin{aligned}\arg z &= \tan^{-1}(2/-2\sqrt{3}) \\ &= 5\pi/6\end{aligned}$$

The Polar form of a complex number

- So far we have plotted the position of a complex number on the Argand diagram by going horizontally on the real axis and vertically on the imaginary.
- This is just like plotting co-ordinates on an x,y axis
- However it is also possible to locate the position of a complex number by the distance travelled from the origin (pole), and the angle turned through from the positive x-axis.
- These are called “Polar coordinates”

The Polar form of a complex number

r is the
MODULUS

The ARGUMENT

(r, θ)

$$\cos\theta = x/r, \sin\theta = y/r$$

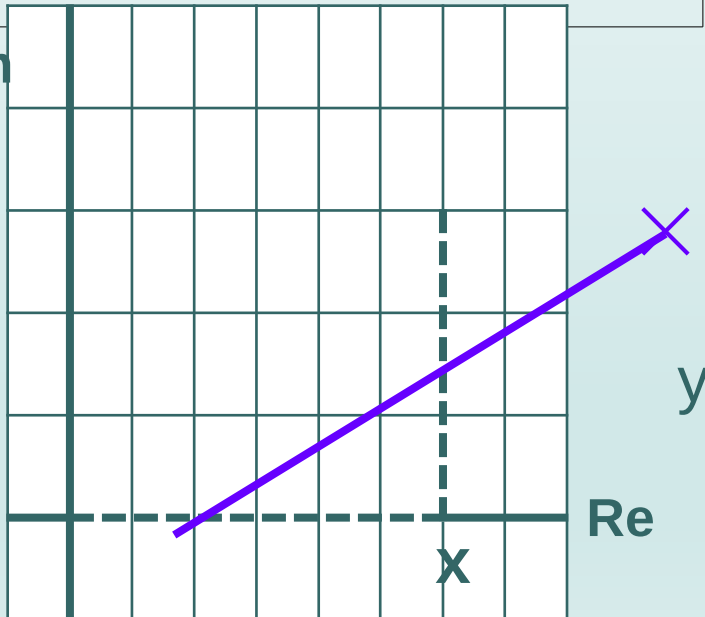
$$x = r \cos\theta, y = r \sin\theta,$$

(x, y)

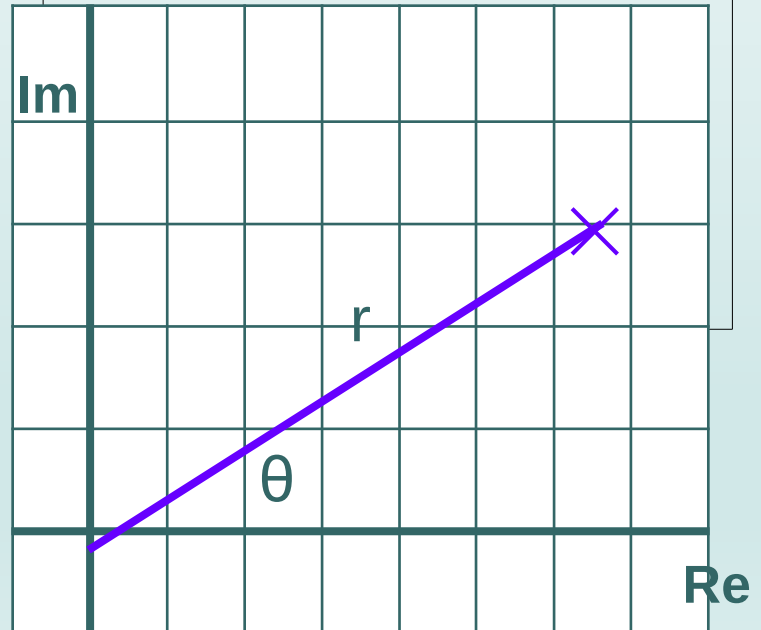
REAL
Part

IMAGINARY
part

Im



9



Converting from Cartesian to Polar

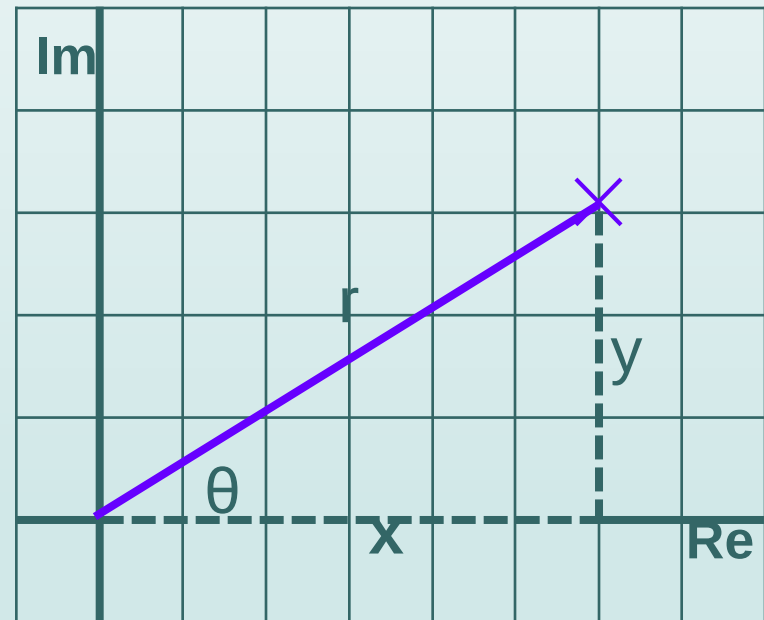
$$(x, y) = (r, \theta) = \left(\sqrt{x^2 + y^2}, \tan^{-1} \frac{y}{x} \right)$$

Convert the following from
Cartesian to Polar

i) $(1, 1) = (\sqrt{2}, \pi/4)$

ii) $(-\sqrt{3}, 1) = (2, 5\pi/6)$

iii) $(-4, -4\sqrt{3}) = (8, -2\pi/3)$



Converting from Polar to Cartesian

$$(r, \theta) = (x, y) (r \cos \theta, r \sin \theta)$$

Convert the following from Polar to Cartesian

i) $(4, \pi/3) = (2, 2\sqrt{3})$

ii) $(3\sqrt{2}, -\pi/4) = (3, -3)$

iii) $(6\sqrt{2}, 3\pi/4) = (-6, 6)$

