

كلية المأمون الجامعة

النظام الدراسي : سنوي
المادة : الاتصالات
الزمن : ساعة ونصف



القسم : هندسة تقنيات الحاسوب
المرحلة : الثانية

للعام الدراسي (2023-2024)

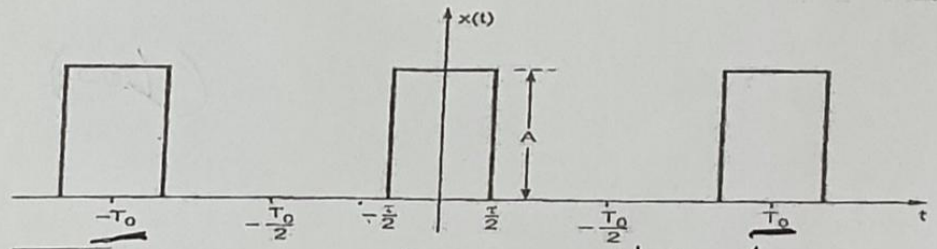
الامتحان نهائي فصل I

Group (a) - Attempt Four (3) Questions only

- Q1) Choice the correct answer (Select five)

- Communication is a process of
 - Signal amplification
 - Signal attenuation
 - Information exchange
- The function of the input transducer is
 - to transmit the message signal
 - to modulate the message signal
 - to convert message signal to electric signal
- Radio signals are made of
 - Voltage and current
 - Electric and magnetic fields
 - Noise and Data
- Periodic signal is
 - power signal
 - Energy signal
 - Power and energy
- In radio transmission, the medium of transmission is
 - Free space
 - Antenna
 - cable
- Fourier series technique use to decompose the signal, when it is
 - Non periodic
 - Periodic
 - Periodic non sinusoidal

Q2) Find Fourier Series expansion for following rectangular pulses $x(t)$ with $A=10$



$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

$$x(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt$$

Q3) 1. Classify the signal, $X(t) = 10 \text{ rect } \{t/T_0\}$

2. Determine whether it is energy signal or power signal

3. Find their normalized energy or power

Q4) a given signal $x(t) = e^{-0.6t}$

1. Classify the signal

2. Find its Fourier Transform, by:

a. Fourier Transform Integral

b. Time scale property

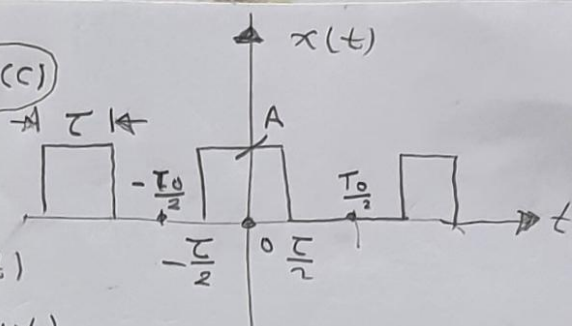
and compare the results

$$(10)^2$$

$$(100)$$

Q.1) 1.(c), 2.(c), 3.(b), 4.(a), 5.(c), 6.(c)
 Q.2)

Fourier series by quadrature



$$i.e.: x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A dt = \frac{A}{T_0} [t]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} = \frac{A}{T_0} \left[\frac{T_0}{2} - \left(-\frac{T_0}{2}\right) \right] = \frac{A\tau}{T_0}$$

or by the area = $\frac{A\tau}{T_0}$

Since the signal is symmetrical about the vertical axis, i.e. $x(-t) = x(t)$, so, in Fourier series representation only cosine terms are present, this means that $b_n = 0$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cdot \cos(n\omega_0 t) dt, \text{ with } x(t) = A \text{ in this range, } \therefore a_n = \frac{2A}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(n\omega_0 t) dt$$

$$= \frac{2A}{T_0} \left[\frac{\sin(n\omega_0 t)}{n\omega_0} \right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}}$$

$$= \frac{2A}{T_0 n\omega_0} \left[\sin\left(n\omega_0 \frac{T_0}{2}\right) - \sin\left(n\omega_0 \left(-\frac{T_0}{2}\right)\right) \right], \text{ with } \sin\left(-\frac{n\omega_0 T_0}{2}\right) = -\sin\left(\frac{n\omega_0 T_0}{2}\right)$$

$$a_n = \frac{2A}{n\omega_0 T_0} [2\sin(n\omega_0 \frac{T_0}{2})] = \frac{4A}{n\omega_0 T_0} \sin(n\omega_0 \frac{T_0}{2})$$

$$\text{with } \omega_0 = \frac{2\pi}{T_0}, a_n = \frac{4A}{n \cdot \frac{2\pi}{T_0} \cdot T_0} \sin\left(n \frac{2\pi}{T_0} \cdot \frac{T_0}{2}\right)$$

$$a_n = \frac{4A}{n \cdot 2\pi} \sin\left(\frac{n\pi T_0}{T_0}\right) = \frac{2A}{n\pi} \sin\left(\frac{n\pi T_0}{T_0}\right)$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

$$= \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi T_0}{T_0}\right) \cdot \cos\left(n \frac{2\pi}{T_0} t\right)$$

(2-4)

We have $C_n = \frac{1}{2}(a_n - j b_n) = \frac{1}{2}(a_n - j \cdot 0) = \frac{a_n}{2}$

$$C_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) / 2 = \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right)$$

and $C_{-n} = \frac{1}{2}(a_n + j b_n) = A$

$$= \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right)$$

Fourier series by Exponential

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}, \quad C_n = \frac{1}{T_0} \int_t^{t+T_0} x(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{T_0} \int_t^{t+T_0} x(t) e^{-jn\omega_0 t} dt \quad \text{Since } \omega = \frac{2\pi}{T_0}$$

$$= \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} A e^{-j\frac{2\pi n t}{T_0}} dt$$

$$C_n = \frac{A}{T_0} \left[\frac{e^{-j\frac{2\pi n t}{T_0}}}{-j\frac{2\pi n}{T_0}} \right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} = \frac{-A}{T_0 \cdot j\frac{2\pi n}{T_0}} \left[\frac{e^{-j\frac{2\pi n \frac{T_0}{2}}}{T_0}}{e^{\frac{j\frac{2\pi n \frac{T_0}{2}}}{T_0}}} \right]$$

$$= \frac{A}{j2\pi n} \left[e^{j\frac{\pi n \tau}{T_0}} - e^{-j\frac{\pi n \tau}{T_0}} \right] = \frac{A}{n\pi} \left[\frac{e^{j\frac{\pi n \tau}{T_0}} - e^{-j\frac{\pi n \tau}{T_0}}}{2j} \right]$$

$$C_n = \frac{A}{n\pi} \sin\left(\frac{\pi n \tau}{T_0}\right)$$

because $\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin(\theta)$

$$C_{-n} = \frac{1}{T_0} \int_t^{t+T_0} x(t) e^{jn\omega_0 t} dt = \frac{A}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{jn\omega_0 t} dt$$

$$= \frac{A}{T_0} \left[\frac{e^{j\frac{2\pi n t}{T_0}}}{j\frac{2\pi n}{T_0}} \right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}}$$

$$= \frac{A}{j2\pi n} \left[e^{j\frac{\pi n \tau}{T_0}} - e^{-j\frac{\pi n \tau}{T_0}} \right] = \frac{A}{n\pi} \left[\frac{e^{j\frac{\pi n \tau}{T_0}} - e^{-j\frac{\pi n \tau}{T_0}}}{2j} \right]$$

$$C_{-n} = \frac{A}{n\pi} \sin\left(\frac{\pi n \tau}{T_0}\right)$$

(3-3)

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn\frac{2\pi}{T_0}t} dt \quad \text{--- } C_n \text{ calculated directly}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) e^{jn\frac{2\pi}{T_0}t}$$

To check, let us calculate a_0, a_n, b_n from above results

$$\begin{aligned} a_0 = C_0 &= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) dt = \frac{1}{T_0} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A dt = \frac{A}{T_0} [t]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \\ &= \frac{A}{T_0} \left[\frac{\tau}{2} - \left(-\frac{\tau}{2}\right) \right] = \frac{A\tau}{T_0} \end{aligned}$$

$$\begin{aligned} a_n = C_n + C_{-n} &= \frac{A}{n\pi} \sin\left(\frac{\pi n\tau}{T_0}\right) + \frac{A}{-n\pi} \sin\left(\frac{\pi n\tau}{T_0}\right) \\ &= \frac{2A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) \end{aligned}$$

$$\begin{aligned} b_n = C_n - C_{-n} &= \frac{A}{n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) - \frac{A}{-n\pi} \sin\left(\frac{n\pi\tau}{T_0}\right) \\ &= 0 \end{aligned}$$

∴ Same Results

Q3)

$$x(t) = B \text{ rect}(t/T_0) \quad (4-4)$$

① The signal is non periodic

② It is Energy signal

$$\begin{aligned} \text{③ Normalized Energy} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt \\ &= \int_{-\infty}^{\infty} x^2(t) dt \\ &= \int_{-T_0/2}^{T_0/2} (B)^2 dt = B^2 [t]_{-T_0/2}^{T_0/2} \\ &= B^2 [T_0] = B^2 T_0 \end{aligned}$$

Q4)

$$x(t) = e^{-0.8t}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$\text{but } x(t) = e^{-0.8t} u(t)$$

$$\begin{aligned} \therefore X(f) &= \int_0^{\infty} e^{-0.8t} e^{-j2\pi f t} dt \\ &= \int_0^{\infty} e^{-(0.8 + j2\pi f)t} dt \\ &= \frac{1}{-(0.8 + j2\pi f)} \left[e^{-(0.8 + j2\pi f)t} \right]_0^{\infty} \\ &= \frac{1}{(0.8 + j2\pi f)} \left[e^{-\infty} - e^0 \right] = \frac{-1}{(0.8 + j2\pi f)} [0 - 1] \\ &= \frac{1}{0.8 + j2\pi f} \end{aligned}$$

Let $a = 0.8$, If $a = 1 \therefore X(f) = \frac{1}{1 + j2\pi f}$

By Time scaling property

$$X(at) = \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\therefore X(0.8t) = \frac{1}{0.8} \cdot \frac{1}{1 + j2\pi \frac{f}{0.8}} = \frac{1}{0.8 + j2\pi f}$$

Q1)