

Parallel DC Circuits

introduction

Introduction

INTRODUCTION

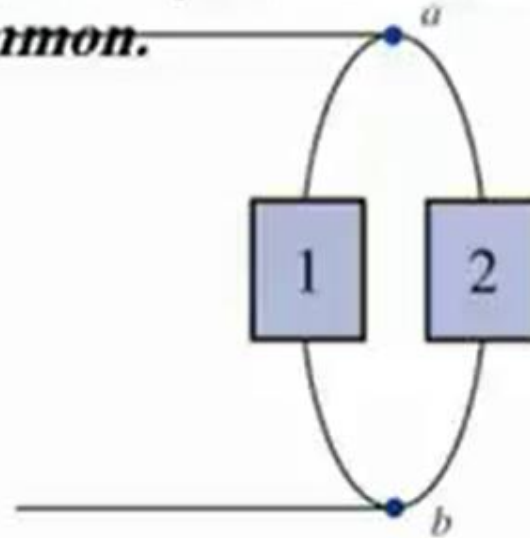
- Two network configurations, series and parallel, form the framework for some of the most complex network structures.
- A clear understanding of each will pay enormous dividends as more complex methods and networks are examined.

Parallel Resistors

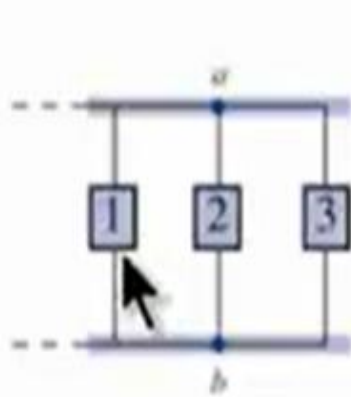
PARALLEL RESISTORS

The term *parallel* is used so often to describe a physical arrangement between two elements that most individuals are aware of its general characteristics.

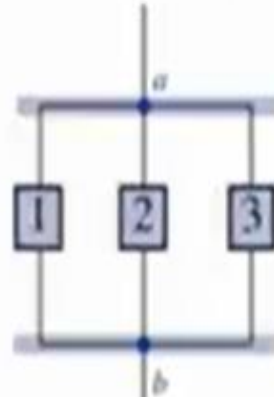
In general, *two elements, branches, or circuits are in parallel if they have two points in common.*



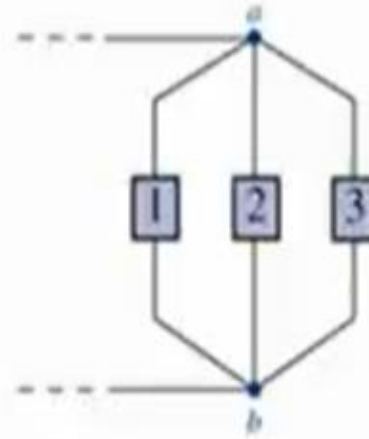
Examples



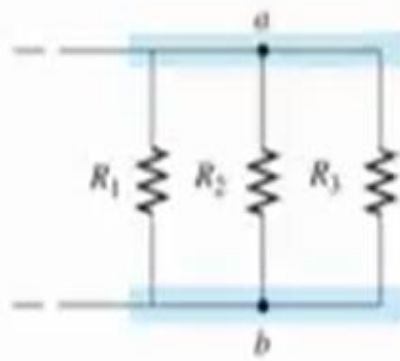
(a)



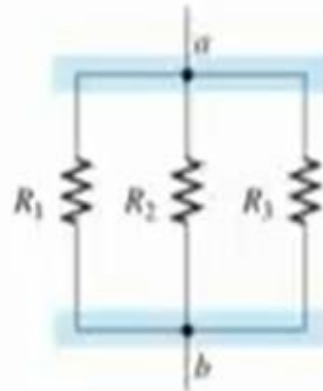
(b)



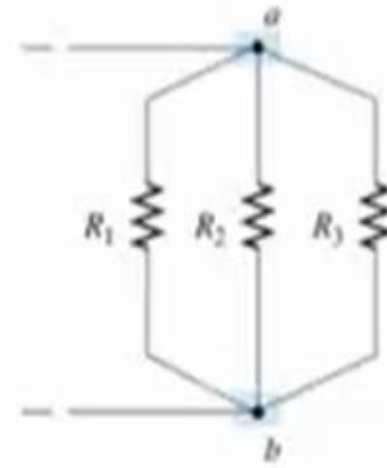
(c)



(a)



(b)

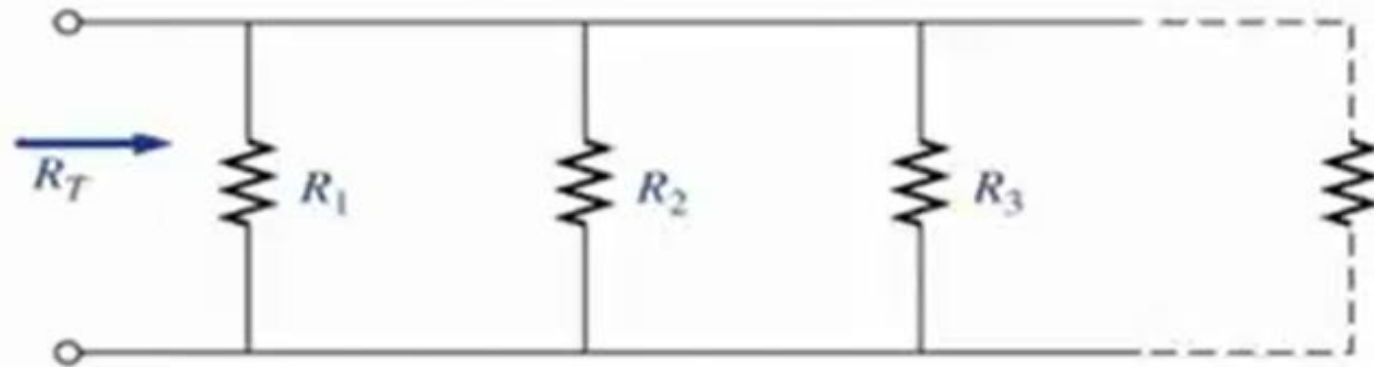


(c)

Total resistance

PARALLEL RESISTORS

For resistors in parallel as shown in Fig. the total resistance is determined from the following equation:



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Examples

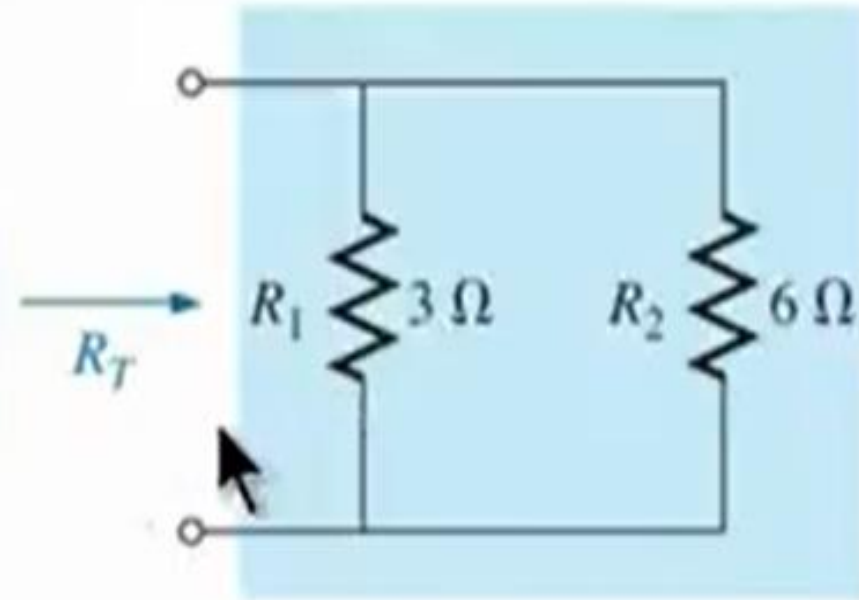
EXAMPLE 1

Find the total resistance of the parallel network in Fig

SOLUTION

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{3\ \Omega} + \frac{1}{6\ \Omega}}$$

$$= 2\ \Omega$$

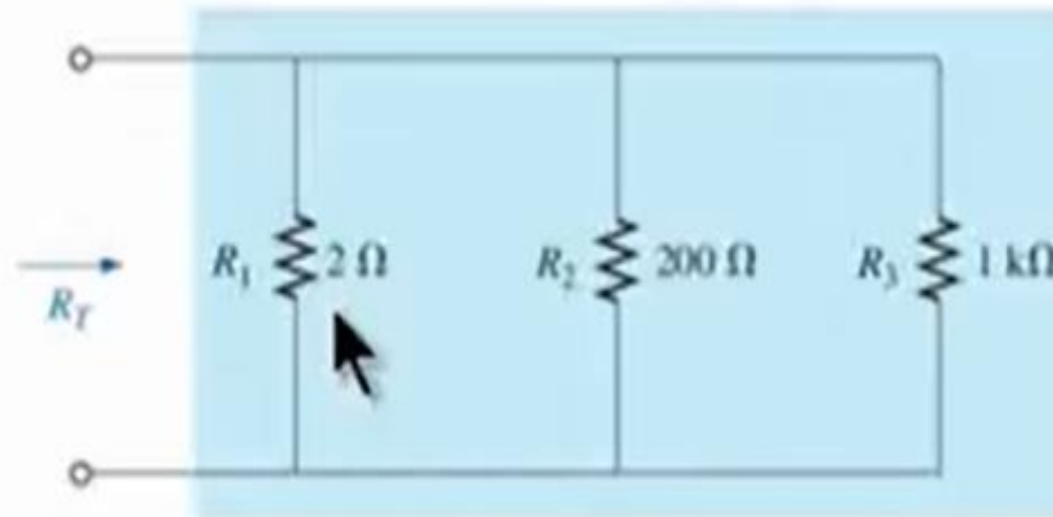


Examples

EXAMPLE 2

Find the total resistance of the parallel network in Fig

SOLUTION



$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{2\ \Omega} + \frac{1}{200\ \Omega} + \frac{1}{1\ \text{k}\Omega}} = 1.98\ \Omega$$

Equal value resistances

PARALLEL RESISTORS

The total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.

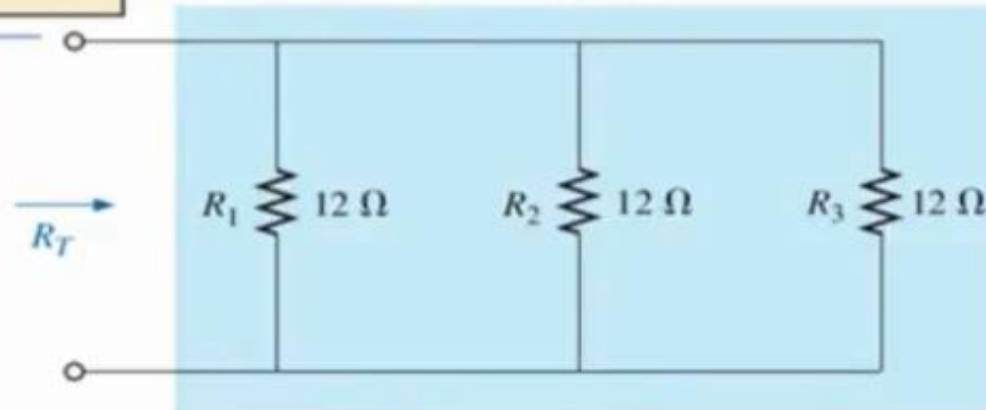
$$R_T = \frac{R}{N}$$

EXAMPLE 3

Find the total resistance of the parallel resistors in Fig.

SOLUTION

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = 4\ \Omega$$



Special case

PARALLEL RESISTORS

Special Case: Two Parallel Resistors

The total resistance of two parallel resistors is simply the product of their values divided by their sum.

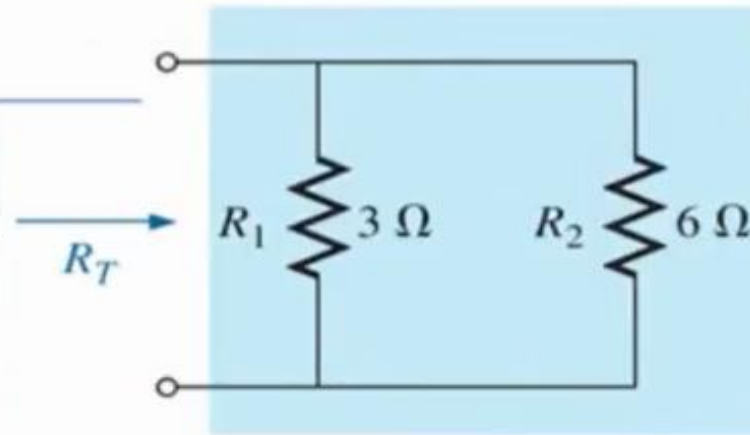
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

EXAMPLE 4

Find the total resistance of the parallel network in Fig

SOLUTION

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3 \Omega)(6 \Omega)}{3 \Omega + 6 \Omega} = \frac{18}{9} \Omega = 2 \Omega$$



Examples

EXAMPLE (5)

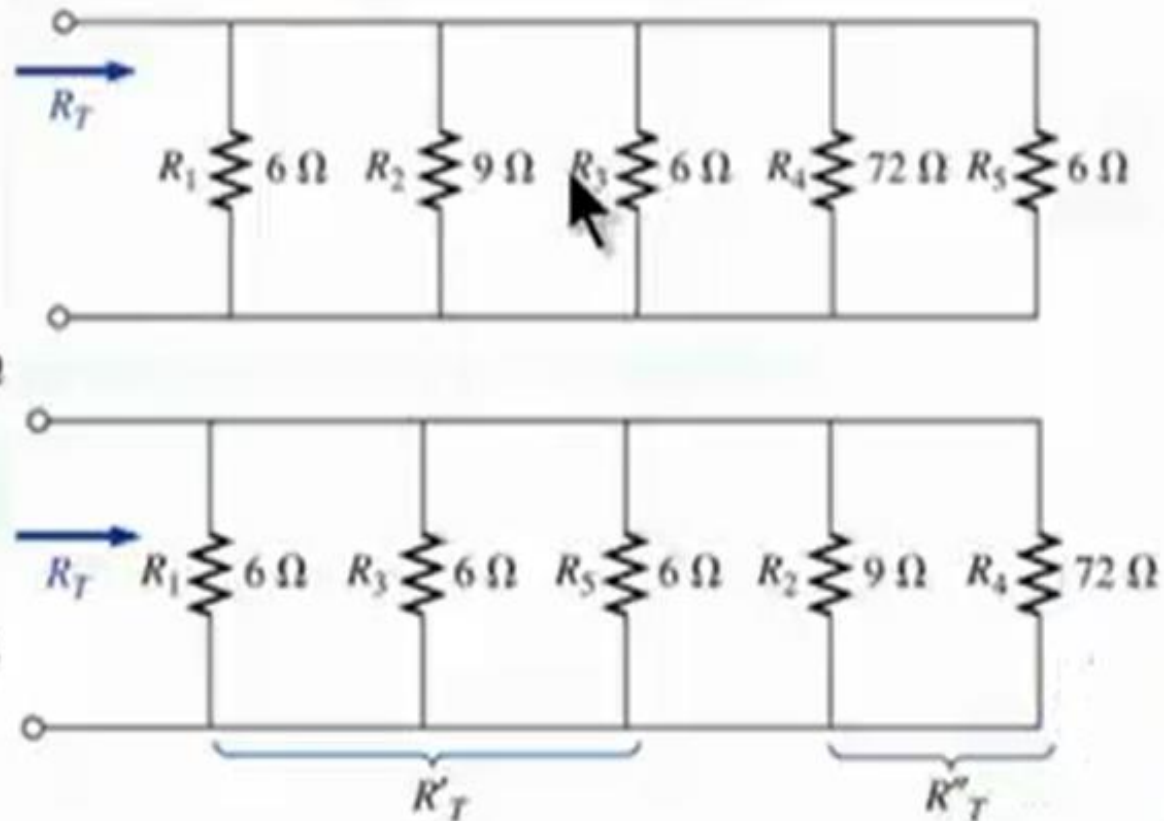
Determine the total resistance of the parallel elements in Fig.

SOLUTION

$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648}{81}\ \Omega = 8\ \Omega$$

$$R_T = \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16}{10}\ \Omega = 1.6\ \Omega$$



Examples

EXAMPLE 6

Determine the value of R_2 in Fig. to establish a total resistance of $9 \text{ k}\Omega$

SOLUTION

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

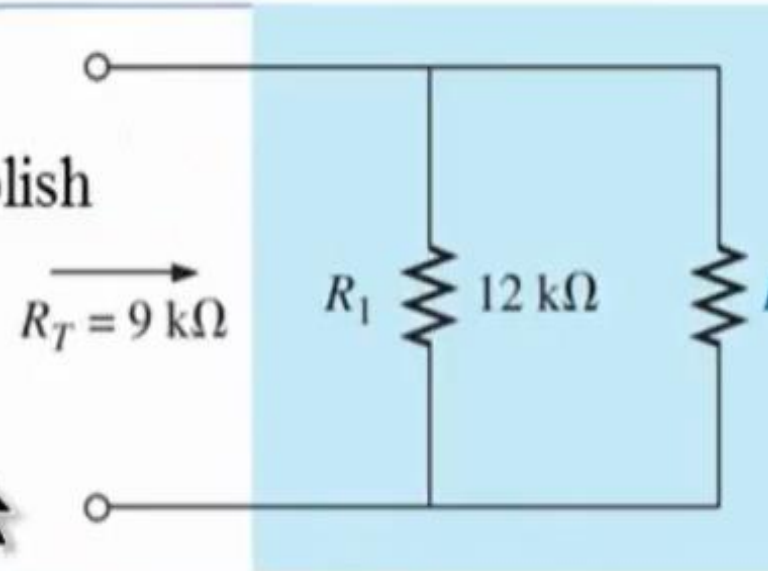
$$R_T(R_1 + R_2) = R_1 R_2$$

$$R_T R_1 + R_T R_2 = R_1 R_2$$

$$R_T R_1 = R_1 R_2 - R_T R_2$$

$$R_T R_1 = (R_1 - R_T) R_2$$

$$R_2 = \frac{R_T R_1}{R_1 - R_T}$$

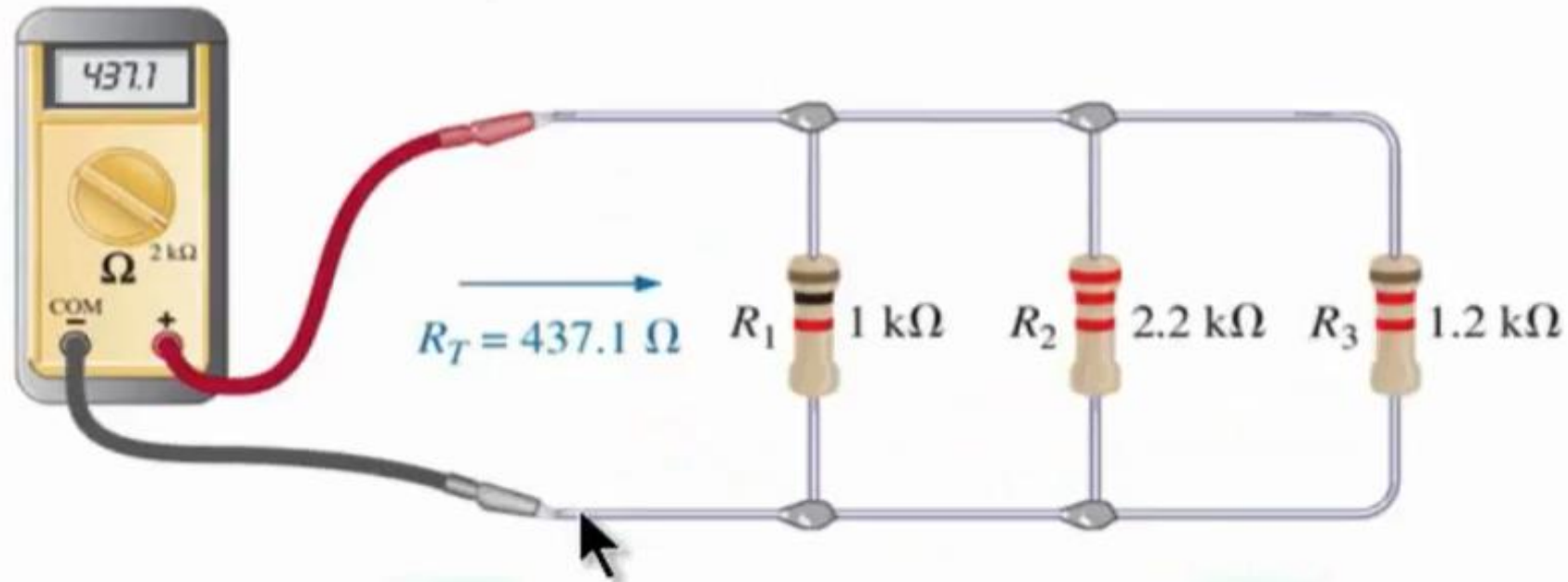


Substituting values gives :

$$R_2 = \frac{(9 \text{ k}\Omega)(12 \text{ k}\Omega)}{12 \text{ k}\Omega - 9 \text{ k}\Omega} = \frac{108}{3} \text{ k}\Omega = 36 \text{ k}\Omega$$

Measuring total resistance in laboratory

PARALLEL RESISTORS



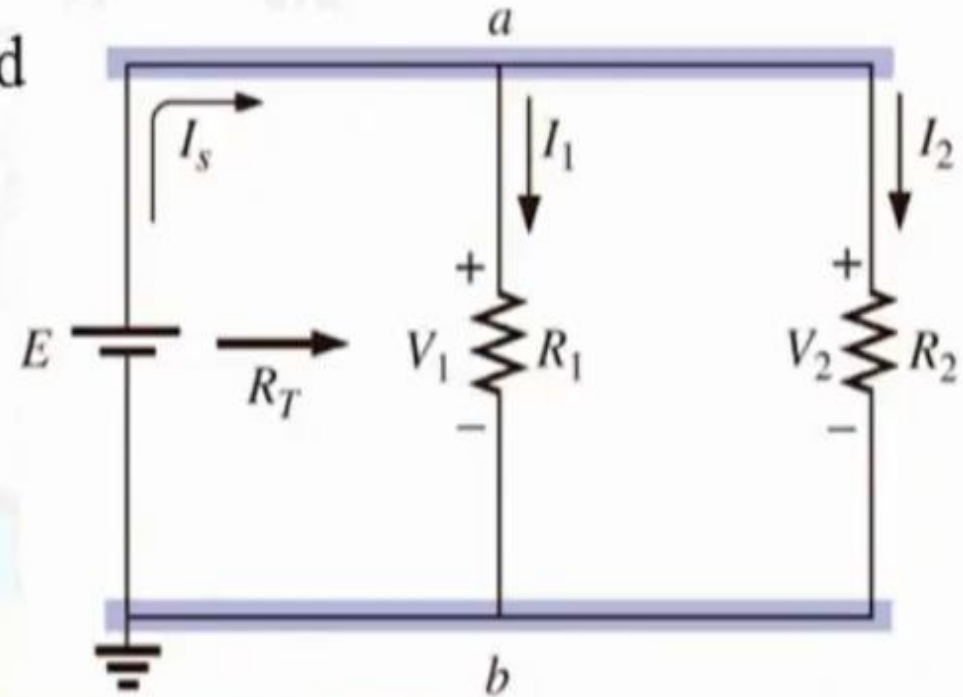
Using an ohmmeter to measure the total resistance of a parallel network.

Continue

PARALLEL CIRCUITS

A parallel circuit can now be established by connecting a supply across a set of parallel resistors as shown in Fig.

The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor.



Voltage across parallel resistors

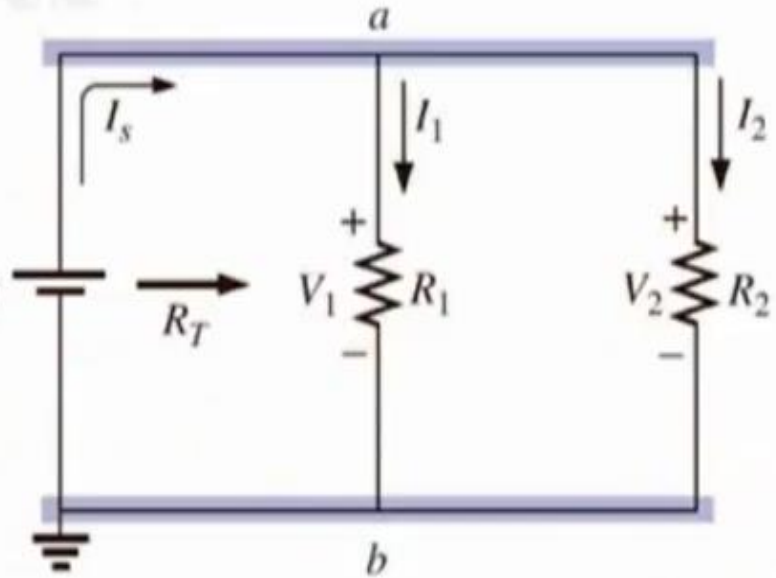
PARALLEL CIRCUITS

In general:

The voltage is always the same across parallel elements.

Therefore, remember that :

If two elements are in parallel, the voltage across them must be the same.



Current in parallel resistors

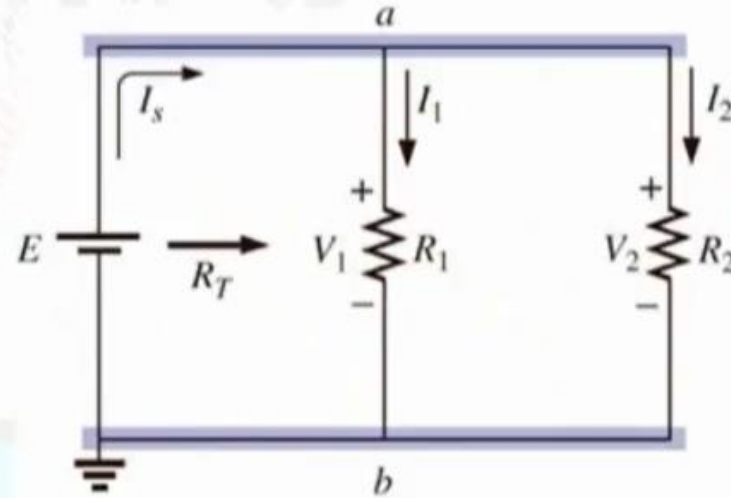
PARALLEL CIRCUITS

$$V_1 = V_2 = E$$

$$I_s = \frac{E}{R_T}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$



for a parallel circuit, the source current equals the sum of the branch currents, while for a series circuit, the applied voltage equals the sum of the voltage drops.

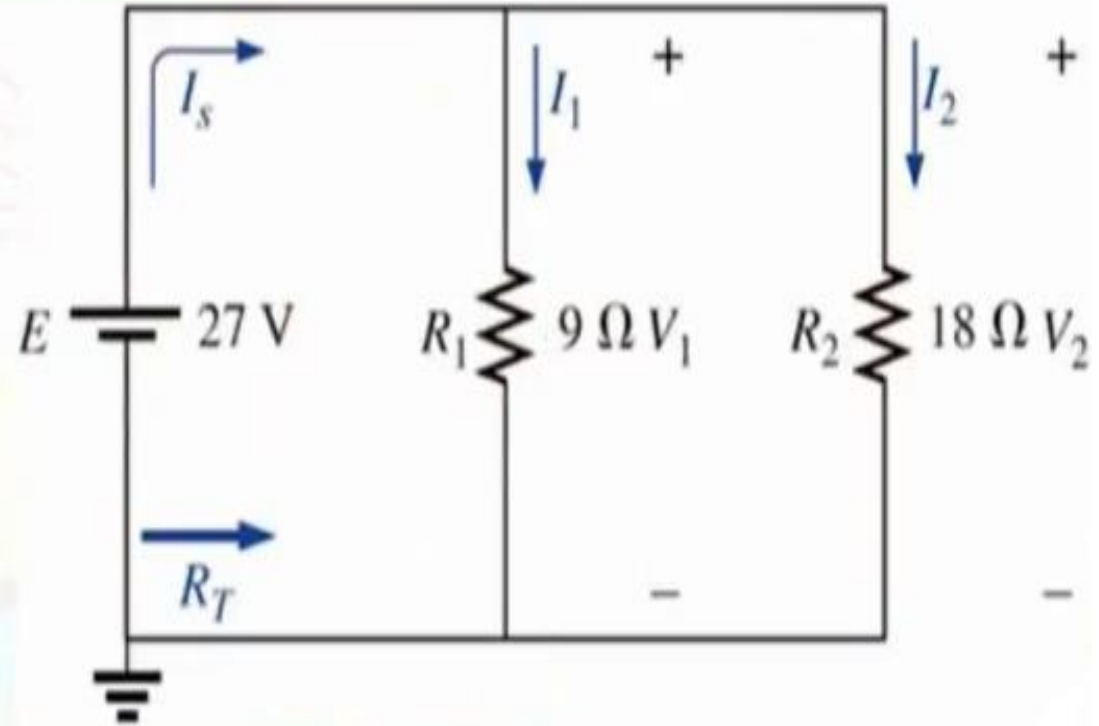
$$I_s = I_1 + I_2$$

Examples

EXAMPLE (6)

For the parallel network in Fig

- Find the total resistance.
- Calculate the source current.
- Determine the current through each parallel branch.



SOLUTION

a.
$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\ \Omega)(18\ \Omega)}{9\ \Omega + 18\ \Omega} = \frac{162}{27}\ \Omega = 6\ \Omega$$

Examples

EXAMPLE (6)

SOLUTION

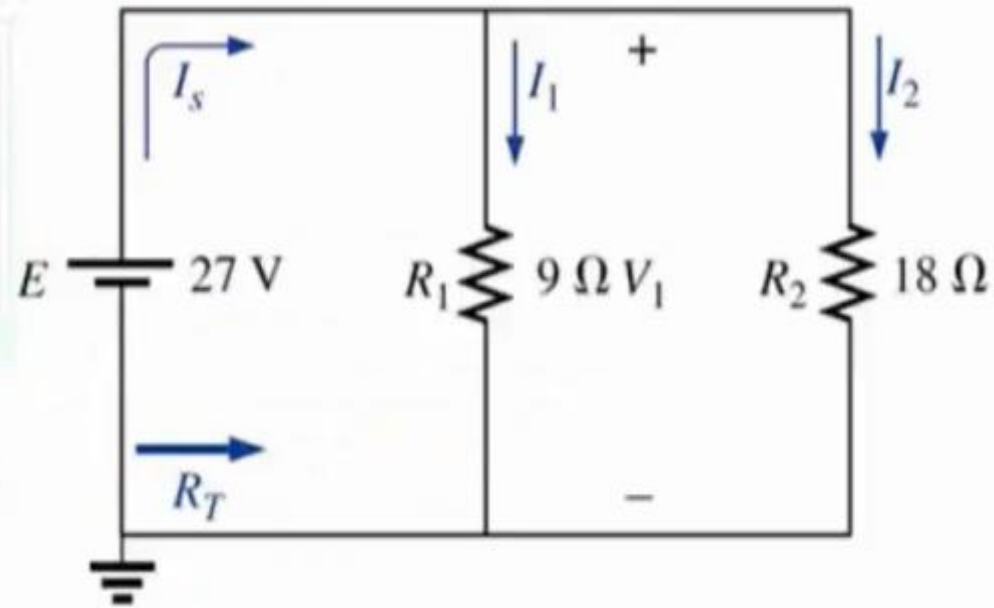
b. Applying Ohm's law gives

$$I_s = \frac{E}{R_T} = \frac{27 \text{ V}}{6 \Omega} = 4.5 \text{ A}$$

c. Applying Ohm's law gives

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27 \text{ V}}{9 \Omega} = 3 \text{ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27 \text{ V}}{18 \Omega} = 1.5 \text{ A}$$



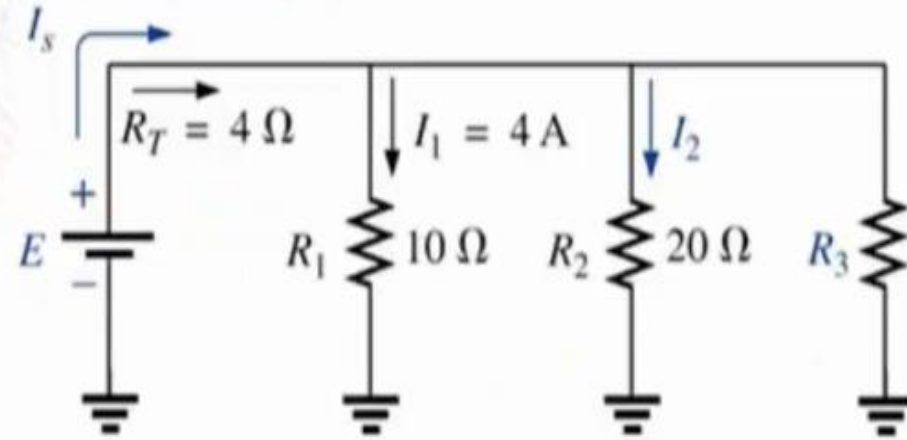
$$I_s = 4.5 \text{ A} = I_1 + I_2 = 3 \text{ A} + 1.5 \text{ A} = 4.5$$

Examples

EXAMPLE (7)

Given the information provided in Fig.

- Determine R_3 .
- Find the applied voltage E .
- Find the source current I_s .
- Find I_2 .



SOLUTION

$$\text{a. } \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4 \Omega} = \frac{1}{10 \Omega} + \frac{1}{20 \Omega} + \frac{1}{R_3}$$

$$R_3 = \frac{1}{0.1 \text{ S}} = \mathbf{10 \Omega}$$

Conclusion

PARALLEL CIRCUITS

Conclusion

The term parallel is used so often to describe a physical arrangement between two or more elements that most individuals are aware of its general characteristics.

In general, two elements, branches, or circuits are in parallel if they have two points in common.

In general,

The voltage is always the same across parallel elements, while the current is divided based on the resistance value.