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*Lecture 4

Boolean Algebra and Logic Simplification

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* Boolean Operations and Expressions

- * Boolean algebra is the mathematics of digital logic. A basic knowledge of Boolean algebra is indispensable to the study and analysis of logic circuits.
- * *Variable, complement, and literal* are terms used in Boolean algebra. A **variable** is a symbol used to represent an action, a condition, or data. Any single variable can have only a 1 or a 0 value. The **complement** is the inverse of a variable and is indicated by a bar over the variable (overbar). **For example**, the complement of the variable A is $\bar{A}$. If $A = 1$, then $\bar{A} = 0$. The complement of the variable A is read as “not A ” or “ A bar”. A **literal** is a variable or the complement of a variable.

* A. Boolean Addition:

* **Boolean addition** is equivalent to the OR operation. In logic circuits, a sum term is produced by an OR operation with no AND operations involved. Some examples of sum terms are $A + B$, $A + \overline{B}$, $A + B + \overline{C}$, and $A + B + C + \overline{D}$.

EXAMPLE 1: Determine the values of A , B , C , and D that make the sum term $A + \overline{B} + C + \overline{D}$ equal to 0.

Solution: For the sum term to be 0, each of the literals in the term must be 0. Therefore, $A = 0$, $\overline{B} = 0$, $C = 0$, and $\overline{D} = 0$ so that $B = 1$, and $D = 1$

$$A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$$

*B. Boolean Multiplication:

* **Boolean multiplication** is equivalent to the AND operation. In logic circuits, a product term is produced by an AND operation with no OR operations involved. Some examples of product terms are AB , $\overline{A}\overline{B}$, ABC , and $\overline{A}\overline{B}C\overline{D}$.

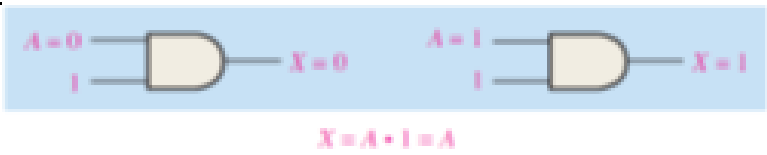
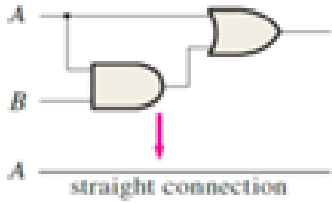
EXAMPLE 2: Determine the values of A , B , C , and D that make the product term $\overline{A}\overline{B}C\overline{D}$ equal to 1.

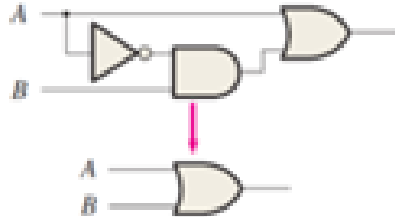
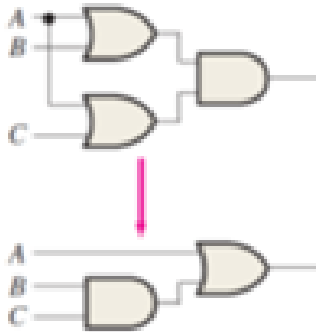
Solution: For the product term to be 1, each of the literals in the term must be 1. Therefore, $A = 1$, $\overline{B} = 1$, $C = 1$, and $\overline{D} = 1$ so that, $B = 0$, and $D = 0$.

$$\overline{A}\overline{B}C\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

*Laws of Boolean Algebra

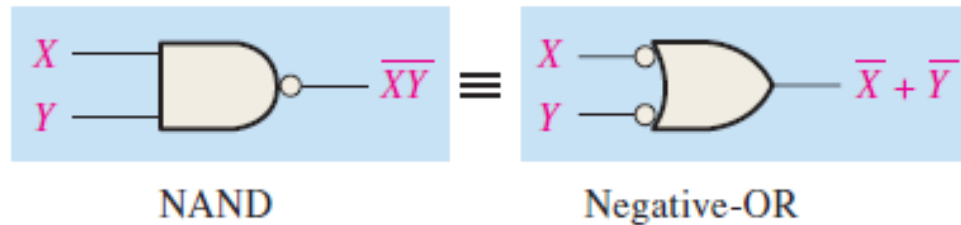
	Rule	Conditions
1	$A + 0 = A$	 $X = A + 0 = A$
2	$A + 1 = 1$	 $X = A + 1 = 1$
3	<u>A</u> $\cdot 0 = 0$	 $X = A \cdot 0 = 0$

4	$A \cdot 1 = A$	 $X = A \cdot 1 = A$
5	$A + A = A$	 $X = A + A = A$
6	$A + \bar{A} = 1$	 $X = A + \bar{A} = 1$
7	$A \cdot A = A$	 $X = A \cdot A = A$
8	$A \cdot \bar{A} = 0$	 $X = A \cdot \bar{A} = 0$
9	$\bar{\bar{A}} = A$	 $\bar{\bar{A}} = A$
10	$A + AB = A$	This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows: $A + AB = A \cdot 1 + AB = A(1 + B)$ Factoring (distributive law) $= A \cdot 1$ Rule 2: $(1 + B) = 1$ $= A$ Rule 4: $A \cdot 1 = A$  straight connection

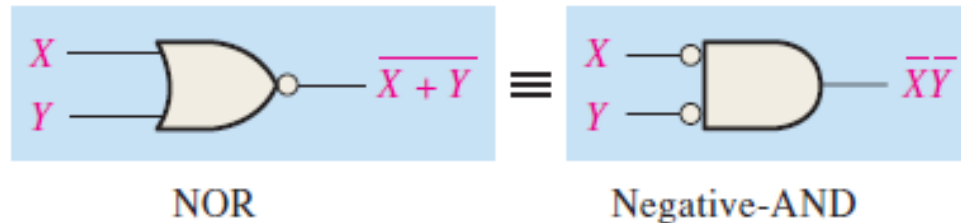
	$A + \bar{A}B = A + B$	This rule can be proved as follows: $ \begin{aligned} A + \bar{A}B &= (A + AB) + \bar{A}B && \text{Rule 10: } A = A + AB \\ &= (AA + AB) + \bar{A}B && \text{Rule 7: } A = AA \\ &= AA + AB + A\bar{A} + \bar{A}B && \text{Rule 8: adding } A\bar{A} = 0 \\ &= (A + \bar{A})(A + B) && \text{Factoring} \\ &= 1 \cdot (A + B) && \text{Rule 6: } A + \bar{A} = 1 \\ &= A + B && \text{Rule 4: drop the 1} \end{aligned} $ 
12	$(A + B)(A + C) = A + BC$	This rule can be proved as follows: $ \begin{aligned} (A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\ &= A + AC + AB + BC && \text{Rule 7: } AA = A \\ &= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\ &= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\ &= A(1 + B) + BC && \text{Factoring (distributive law)} \\ &= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\ &= A + BC && \text{Rule 4: } A \cdot 1 = A \end{aligned} $ 

*DeMorgan's Theorems

- * DeMorgan, a mathematician who knew Boole, proposed two theorems that are an important part of Boolean algebra



Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X} \overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

* Standard Forms of Boolean Expressions

* All Boolean expressions, regardless of their form, can be converted into either of two standard forms: The **Sum-Of-Products (SOP)** form or the **Product-Of-Sums (POS)** form.

* **The Sum-of-Products (SOP) Form:** A product term was a term consisting of the product (Boolean multiplication) of literals (variables or their complements). When two or more product terms are summed by Boolean addition, the resulting expression is a **sum-of-products (SOP)**. Some examples are:

$$AB + ABC$$

$$ABC + CDE + \overline{BCD}$$

$$\overline{AB} + \overline{AB}\overline{C} + AC$$

Also, an SOP expression can contain a single-variable term, as in $A + ABC + BCD$.

For Example: $\overline{\overline{A + B} + C} = \overline{\overline{A + B}}\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$

* **The Product-of-Sums (POS) Form:** A sum term was a term consisting of the sum (Boolean addition) of

literals (variables or their complements). When two or more sum terms are multiplied, the resulting expression is a **product-of-sums (POS)**. Some examples are:

$$(\bar{A} + B)(A + \bar{B} + C)$$

$$(\bar{A} + \bar{B} + \bar{C})(C + \bar{D} + E)(\bar{B} + C + D)$$

$$(A + B)(A + \bar{B} + C)(\bar{A} + C)$$

A POS expression can contain a single-variable term, as in $\bar{A}(A + \bar{B} + C)(\bar{B} + \bar{C} + D)$.

EXAMPLE : Convert the following SOP expression to an equivalent

$$\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + ABC$$

Solution:

The evaluation is as follows: 000 + 010 + 011 + 101 + 111

Inputs			Output	Sum Term
A	B	C	X	
0	0	0	1	
0	0	1	0	$A + B + \bar{C}$
0	1	0	1	
0	1	1	1	
1	0	0	0	$\bar{A} + B + C$
1	0	1	1	
1	1	0	0	$\bar{A} + \bar{B} + C$
1	1	1	1	